

E-ELT PROGRAMME

MICADO Phase A

Top level Data Reduction User Requirements

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ABBREVIATIONS AND ACRONYMS

AO	adaptive optics
CAD	computer aided design
CAE	computer aided engineering
ECSS	European Cooperation for Space Standardization
E-ELT	European Extremely Large Telescope
ESO	European Southern Observatory
FDR	Final Design Review
FTE	Full Time Equivalent (year)
GLAO	ground layer adaptive optics
GMT	Giant Magellan Telescope
JWST	James Web Space Telescope
LESIA	Laboratoire d'Etudes Spatiales et Instrumentations pour l'Astrophysique
LTAO	laser tomography adaptive optics
MAIT	Manufacture, Assembly, Integration, Test
MAORY	Multi-conjugate Adaptive Optics Relay
MCAO	multi-conjugate adaptive optics
MICADO	Multi-adaptive optics Imaging Camera for Deep Observations
MPE	Max-Planck-Institut für extraterrestrische Physik
MPIA	Max-Planck-Institut für Astronomie
NOVA	Nederlandse Onderzoekschool voor Astronomie
OAPD	Osservatorio Astronomico di Padova
PAE	Preliminary Acceptance in Europe
PAO	Preliminary Acceptance at the Observatory
PA/QA	Product Assurance / Quality Assurance
PDR	Preliminary Design Review
PSF	Point Spread Function
RTD	Real Time Display
SCAO	single-conjugate adaptive optics
TMT	Thirty Meter Telescope
USM	Universitäts-Sternwarte München
WP	Workpackage

1 SCOPE

This document describes the top level user requirements for the data reduction of MICADO observations. It lays down the requirements on the data processing infrastructure and data reduction procedures required to achieve the MICADO science goals. The requirements also involve the observational operations plan, calibration plan, and the technical design of MICADO, MAORY and E-ELT. An important aim of the document is to identify which requirements can be fulfilled by implementing well-known existing approaches and solutions and which requirements require further research and/or development.

2 APPLICABLE AND REFERENCE DOCUMENTS

2.1 Applicable Documents

The following applicable documents form a part of the present document to the extent specified herein. In the event of conflict between applicable documents and the content of the present document, the applicable document shall be taken as superseding.

- AD1 Common definitions and acronyms , E-ESO-SPE-313-0066, Issue 1
- AD2 E-ELT Interfaces for Scientific Instruments, E-TRE-ESO-586-0252, issue 1
- AD3 Call for Proposal For a Phase A Study of a High Angular Resolution Camera for the E-ELT, Specifications of the Instrument to be studied, E-ESO-SPE-561-0097, v1.0
- AD4 Statement of Work for the Phase A Design of MICADO, E-SOW-ESO-561-0127, v1.0

2.2 Reference Documents

- RD1 MICADO Instrument Development and Management Plan, E-PLA-MCD-561-0020, v1.0
- RD2 MICADO Scientific Analysis Report, E-TRE-MCD-561-0007, v2.0
- RD3 MICADO System Overview, E-TRE-MCD-561-0009, v2.0
- RD4 MICADO Design Trade-Off and Risk Assessment, E-TRE-MCD-561-0010, v2.0
- RD5 MICADO Opto-Mechanical Design and Analysis, E-TRE-MCD-561-0011, v5.0
- RD6 MICADO Photometric Study, E-TRE-MCD-561-0023, v1.0
- RD7 MAORY Phase A Study Status at Midterm Review Progress Meeting E-TRE-INA-528-0021 issue 1

3 PROJECT OVERVIEW

MICADO is the Multi-AO Imaging Camera for Deep Observations, which has been designed to work with adaptive optics on the European Extremely Large Telescope. The instrument will image and obtain long-slit spectra at the diffraction limit of the E-ELT. The imaging will be done through both broad- and narrow-band near-infrared filters with a wide (up to nearly 60'') field of view. MICADO is primarily intended to work with the multi-conjugate AO system MAORY; but also, as far as is reasonably possible, it will be designed to work with other AO systems, and includes its own SCAO module.

4 INTRODUCTION

4.1 Scientific requirements

The MICADO science drivers are discussed fully in the Scientific Analysis document (RD2). The primary science drivers are re-iterated here for local reference. MICADO can address a wide range of astrophysical topics using its relatively wide field, high resolution and high sensitivity:

- The environment and internal structure of galaxies and AGN at high redshift
- Star formation history of local galaxies through studies of spatially resolved stellar populations;
- Stellar orbits close to the central massive black hole in the Galactic Centre which test the strong regime of gravity.
- Internal kinematics in nearby large and dwarf galaxies to constrain formation and evolution of galactic nuclei and dark matter content respectively.
- Internal motions in star clusters to detect intermediate black holes and cluster orbits to constrain the formation and evolution of the Galaxy.

The science trade-off and the detailed science cases described in the Science Analysis document (RD2) have led to the baseline science requirements given in Table 1.

Table 1: summary of baseline science requirements for data reduction

<i>Requirement</i>	<i>Baseline Design</i>	<i>Comment</i>
Field of View	$\leq 60''$	Smaller fields sufficient for several science drivers
Spatial Sampling	3mas $\leq 2\text{mas}$	primary arm auxiliary arm, for crowded fields
Total Wavelength Coverage	0.8-2.4 μm	

Throughput	>60%	
Instrumental Distortions	stable	Instrumental distortion variations should not be a dominant contributor to astrometric error budget
Number of Filters	≥ 20 per arm	Both broad- and narrow-band filters
Image Quality	70% Strehl at $1\mu\text{m}$	
Photometric Accuracy	0.03mag	In units of instrumental magnitude system
Astrometric Accuracy	$50\mu\text{as}$	Relative accuracy across the full field of a single carefully calibrated exposure
Spectroscopy	R~3000	

4.2 System overview and observing configurations

The MICADO System Overview (RD3) gives the full instrumental description. Here we summarize characteristics relevant for the data reduction.

The camera consists of reflective optics (except for the ADC, entrance window and filters) that images a contiguous field, having a ~ 75 arcsec diagonal size, onto a focal plane. The ADC is located inside the MICADO cryostat. The focal plane of the primary arm is tiled with detectors, and provides 16000×16000 pixels at a scale of 3mas/pixel . As a baseline we adopt the HAWAII-4RG (with a $15\mu\text{m}$ pixel size), which has been developed to meet the stringent requirements of space astrometry. The optics are a fixed monolithic entity. The focal plane array is on a single fixed mount. The instrument has a gravity invariant rotation for optimal astrometric stability. The design allows for a large wheel with space for 20 filters. Both the filter wheels, as well as the focal plane mechanism, are supported and driven at their rim.

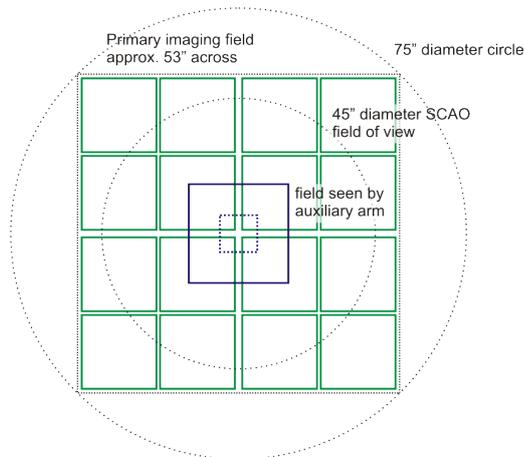


Figure 1: illustration showing how the focal plane will be divided in MICADO. The primary imaging field is approximately $53'' \times 53''$ and will be imaged at a scale of 3mas/pixel by an array of 4×4 HAWAII-4RG detectors. The auxiliary arm has a FoV which is adjacent to the primary one with a FoV.

In addition to the primary imaging arm, MICADO has an auxiliary arm which observes a field adjacent to the primary field. This arm has a separate filter wheel, providing space for an additional 20 filters, and requires only 1 additional HAWAII-4RG detector. In the current design, the auxiliary arm provides 2 important complementary capabilities. First, it can image with 1.5 and 4mas pixel scales over a 6''x6'' and 16''x16'' field of view respectively. The 1.5mas pixel scale is crucial for accurate astrometry in the most extremely crowded fields. Second, slit spectroscopy can be obtained using a 4mas pixel scale and R~3000-5000 grisms. The slit mask is located in the input focal plane mask.

For the requirements described in this document it is assumed that imaging will be taken through broad- and narrow-band filters with central wavelengths in the range 1.0-2.5micron. The total spectral range is assumed to span 0.8-2.35 micron.

MICADO's photometric and astrometric performance will depend on careful calibration. There will be internal calibration sources, such as an integrating sphere for photometry (flatfields) and discharge tubes for spectroscopic work (wavelength calibration). High precision astrometry requires special calibrations using an accurate Hartmann screen (aka 'calibration mask') placed in the camera focal plane to allow long term monitoring and calibration of the pixel scale and field distortions. This is particularly important since there will inevitably be discontinuities between the detectors in the MICADO primary focal plane (both translational and rotational), and because the MAORY field distortions will rotate with respect to those of MICADO.

All the above calibration sources will be located inside a dedicated calibration unit, which will be mounted in front of the AO system at the input focal plane. This is particularly necessary for the distortion calibration, which must be done here because (i) the focal plane is very nearly flat, and (ii) it is then possible to calibrate the MAORY distortions.

5 REQUIREMENTS ON DATA REDUCTION INFRASTRUCTURE

5.1 Overview MICADO data flow

ESO will define the eventual framework for data processing and reduction for E-ELT and its instruments. Here our baseline assumed MICADO data flow and processing is given.

Once the raw MICADO data have been created at the E-ELT site, raw data and first quality control metadata will flow through ESO-DMOD (archive and possibly processing) and subsequently to MICADO processing units and to the MICADO user communities. MICADO processing units are those entities that process MICADO data. The MICADO user communities consist of those entities that use data products from MICADO, but perform no processing. Both the processing units and user community are assumed to be within the astronomical community which can include ESO, research teams at universities and institutes, the Virtual Observatory and data centers, single astronomers. For MICADO data reduction the following data classes are defined:

- Science data: pixel data and metadata containing information on the science targets or their sky frames (if sky offset positions are used),
- Calibration data: pixel and metadata required to remove the instrumental signature from the science data and to photometrically/astrometrically/flux-calibrate/wavelength-calibrate the science data.

For successful reduction and quality control of MICADO data 3 data levels are needed:

- L0. Raw data: all science and calibration data as read out from the detectors and the associated metadata
- L1. Instrumental-calibrated data: science and calibration data from which the instrumental signature has been removed.
- L2. Science-calibrated data: photometrically-/astrometrically-/flux-/wavelength-calibrated science data including stacks and coadditions of these.

The type of data within each data class in each data level is listed in Table 1a for imaging and in Table 1b for spectroscopy.

The 3 data levels are linked by 2 processing chains:

- L0→L1: processing from raw to instrumental-calibrated data
- L1→L2: processing from instrumental-calibrated data to science-calibrated data

Processing units will operate one or more processing chains and entities of the user community will receive data at one or more process levels. It is assumed that the ESO archive will contain at least the raw data and possibly instrumental- and science-calibrated data. Figure 1 shows how data at different process levels is transferred among these entities.

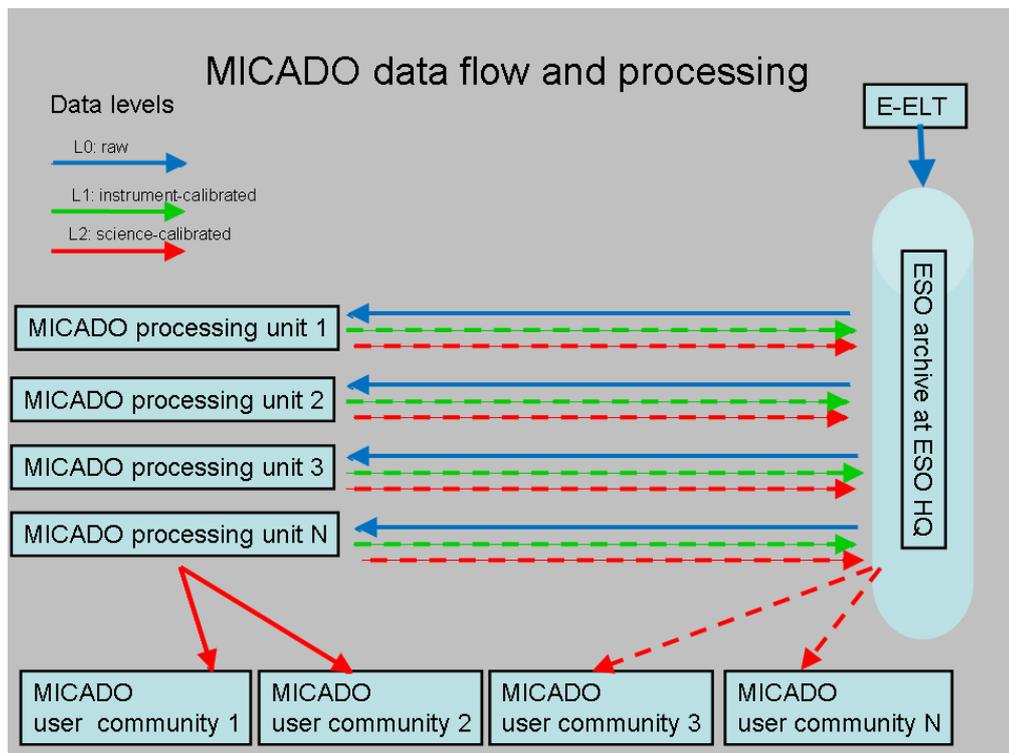


Figure 2: Anticipated transfer and processing of the 3 levels of MICADO data between ESO archive, processing units and user communities. Each transfer of each data level is represented by a colored arrow. Solid/dotted arrows indicate expected/tentative data transfers. Data transfer and processing should be able capable of dealing with nightly observational data rates of up to ~6 Terabytes of data.

The entities range from single users to large data centers or institutes and thus have diverse locations, hardware resources and operating platforms.

Note that documentation is considered part of the infrastructure. It shall contain:

- take-up manual, describing how to implement the data reduction architecture from installation of the infrastructure until production of final products of data reduction
- Description of implemented routines

In the following the top-level requirements are given for the infrastructure required for handling the MICADO data reduction.

Two aspects related to the MICADO imaging science cases drive requirements on the data reduction infrastructure beyond the more common aspects:

- High data rates (~6 TByte/night) for the primary arm imaging due to the combination of short exposures and 16 detectors.
- Maintaining a long-term stable observing system to obtain high precision astrometry for many years.

The MICADO spectroscopic mode can be handled quite well with today's technology and common procedures.

5.2 Data rates and volumes

The maximum data rates will be obtained with imaging using the 16 detectors in the Primary Arm. A single read-out for the Primary Arm imaging is saved to single-precision floating points resulting in a $(16K)^2 \times 4\text{Byte} = 1\text{GByte}$ image. The typical maximum data rate for MICADO corresponds to a full night of imaging assuming a long night of 8 hours science observing. This data rate is computed assuming the following numbers:

- Acquisition time per 1 hour OB: 300 sec
- Read-out time: 1 sec
- Overhead small dither: 2 sec
- Overhead large dither: 20 sec
- Integration time per read-out: 3 sec

We assume a sequence of 10 pointings in a small dither pattern with 5 read-outs per pointing after which a large dither is applied. Within a 1hour OB this sequence can be repeated 14 times: $14 \times [(3\text{s exposure} + 1\text{s read-out}) \times 5 \text{ repeats} + 2\text{s overhead small dither}] \times 10 \text{ repeats} + 20\text{s overhead large dither}$.

The result is 700 detector read-outs per hour yielding 58% observing efficiency. This results in 5.6 TB of raw science data per 8 hours science observing. The sum of night time and day time raw calibration data is estimated at 0.3 TB. Thus in total MICADO acquires ~6 TByte of raw observational data per night. This is 1 to 2 orders of magnitude more than from the current largest data producers at Paranal: VISTA/VIRCAM is about a factor 25 smaller and

VST/OmegaCAM a factor 75 (priv. comm. M. Neeser). Moore's law, if continued will yield more than a factor 300 in storage/computing in 7 years (2010-2016).

Processing to data level 1 requires weight frames for each science frame. Processing to data level 2 will result in stacked data in addition to fully calibrated individual science exposures. The result is an approximate factor of 2 bigger science data volumes at both data level 1 and 2 compared to the corresponding raw data (level 0). If all data at all levels are stored this results in $5 \times 6 = 30$ Tbyte / night, which corresponds to 10 Petabyte / year.

The nightly data volumes are summarized in Table 2. For imaging with the Auxiliary Arm the corresponding data volumes are $1/16^{\text{th}}$ as the image consists of only 1 detector frame. Data volumes for spectroscopy will be negligible compared to the Primary Arm imaging.

As indicated in Table 2, a MICADO archive which contains science and calibration data at data levels 0 and 1 is expected to aggregate at least several Petabyte per year.

Requirement: the entities involved in the MICADO science processing shall be able to handle an observational data rate of ~ 6 Terabyte / night. This means that at the E-ELT storage and processing for quality control purposes shall be able to deal adequately with such a rate. For example, processing time shall not be the time limiting factor for zero and first level QC at E-ELT and/or ESO-DMOD. Also the data transfer between E-ELT and archive shall keep up with this rate. Provided Moore's law holds, the data transfer via at least devices such as USB disks or their equivalents, the long-term storage and daily processing of these nightly volumes will be feasible by 2018. Processing units for MICADO science data shall be able to process data volumes of 6 Terabyte per observing night requiring data storage of up 30 Terabyte for each observing night.

Requirement: Entities involved in MICADO calibration processing shall be able to handle observational data rates of ~ 0.3 Terabyte / night.

A reduction in data volume is desirable to maximize operation speeds and to minimize storage costs. Volume reduction can be achieved by on-detector addition and (does not exit on floats) compression techniques. Possibly lossy detection techniques (e.g., compressed sensing) need to be investigated.

Requirement: Planning and costing of reduction of MICADO data volumes shall be investigated. We note that co-adding frames in the detector control system is one option; and the use of narrow band filters or OH suppressing filters will inevitably lead to longer exposure times, and hence also reduce the total data volume.

Requirement: Data products (pixel data, headers, metadata, catalogues) shall be compliant to the standard requirements by ESO and the Virtual Observatory for the purpose of archiving and distribution.

Table 2: Data items for Imaging. Each column represents a data level and lists the data items created at that data level. They are split in science and calibration classes.

	L0	L1	L2
	Raw	Instrumental-calibrated	Science-calibrated, coadded
Science	Targets Sky-offsets	Targets	Targets
Calib	Health-check data Twilight/artificial Flats Darks Photometric standards Astrometric standards Calib. Mask obs.	Masterflats Darks Astrom fields Photom fields Calib. Mask obs.	Parameters detectors:gains, readnoise Photometric solution Illumination Correction Astrometric solutions

Table 3: Data items for spectroscopy. Each column represents a data level and lists the data items created at that data level. They are split in science and calibration classes.

	L0	L1	L2
	Raw	Instrumental-calibrated	Science-calibrated
Science	Target spectra Sky spectra	Target spectra	Target spectra
Calib	Health-check data Flats Darks Arcs Telluric/flux standards	Masterflats Darks Telluric/flux standards	Detector parameters: gain, readnoise,... Wavelength solution Telluric/flux standards

Table 4: Maximum nightly data volumes for MICADO (imaging with the primary arm). The table assumes 1 night of 8hours of science observing and its associated calibrations. Each cell lists the number of images on top and its corresponding volume on the bottom. Images consist of 16 detector frames for the Primary Arm. For the Auxiliary Arm imaging the corresponding the number of images is the same while the volumes are 1/16th as the image consists of 1 detector frame. Data volumes for spectroscopy are negligible compared to Primary Arm imaging.

	L0 Raw	L1 Instrumental-calibrated	L2 Science-calibrated	Total
Science	5600 5.6TB	11200 11.2TB	~11200 11.3TB	28000 28TB
Calib	270 0.3TB	<<270 <<0.3TB	15000 <<1TB	~16000 <1TB
Total	6000 6.0TB	~12000 ~11.5TB	~27000 ~11.5TB	~44000 ~29TB

The science cases that drive the stringent requirements on astrometry will observe targets at multiple epochs spanning up to many years. Continuous monitoring of all parameters of the E-ELT+MAORY+MICADO system that (might) affect the astrometric accuracy will be essential for such programs. For example, trend analysis on astrometric solutions as a function of these parameters might lead to new insights increasing the astrometric accuracy. This requires a data reduction environment which efficiently traces data lineage. (As will be discussed later on, it needs to be investigated if this also requires an astrometric calibration plan set by monitoring requirements instead of the science observations of that particular night.)

Requirement: The MICADO data reduction infrastructure (software and hardware) shall be capable of monitoring the E-ELT+MAORY+MICADO system in all aspects that can affect the astrometric accuracy.

6 USER REQUIREMENTS FOR DATA REDUCTION: IMAGING

The MICADO image pipeline will be based on near-infrared image reduction pipelines such as ISAAC and NACO at the VLT, VIRCAM at VISTA. Many steps in the data reduction are very similar to those of other multi-detector near-infrared imagers. MICADO data reduction differs from other instruments in two respects: i) the usage of MCAO, possibly preceded by other forms of AO, and ii) the extremely high astrometric accuracy required. The description of the data reduction requirements will therefore focus on requirements related to these topics and pay brief attention to well-established concepts and approaches in near-infrared image reduction. Each section describes a requirement of the image data reduction. The objective of the requirement, its required accuracy and constraints on the requirement are given. Frequency/timing describes how often and when the requirement shall be fulfilled. Feasibility

is defined as “standard reduction” if the requirement can be fulfilled by standard well-established procedures and algorithms. If not, a description is given of required further analysis and/or procedure development.

6.1 Readnoise

Objective: Measure the detector read noise as a standard health check. Pairs of zero-second bias exposures or a combination of frames illuminated by calibration lamp and darks can be used.

Accuracy/constraints: accuracy should be in accordance with the lab values.

Frequency/timing: daytime, daily, health check instrument

Feasibility: standard reduction

6.2 Electronic Ghosts (Cross Talk)

Images from one detector channel may produce secondary images (ghosts) on other channels either positive or negative in sign and may also even cross from one detector to another. In a stable environment, it is feasible to measure the contribution of cross talk from one channel to another by using bright point-like sources, and thereby define a comprehensive crosstalk matrix. Since this is environment specific, determining the final form of this matrix shall be one of the commissioning tasks, and earlier laboratory-based measurements might be used to characterise its likely impact and to investigate ways of minimising the effect.

Objective: Determine the cross-talk matrix.

Accuracy/constraints: The MICADO environment shall be stable so that the contribution of crosstalk from one channel to another is stable over time. Cross-talk terms should be small (i.e. <1%) so that a simple single-pass additive correction scheme can be used to correct for this problem. In addition to on-sky observations the calibration mask can be used. The calibration mask should cover the full detector plane to map the cross-talk amplitude. Lab results on cross-talk will give the requirement on number of holes in the mask and whether the calibration mask shall be movable to sample the detector fully (i.e., dithered observations).

Frequency/timing: daytime, frequency to be determined during commissioning.

Feasibility: standard reduction. The illuminated calibration mask can help to measure cross-talk comprehensively using well-established routines.

6.3 Linearity

Objective: In each frame resulting from a destructive read-out, correct the count of each pixel for non-linearity.

Accuracy/constraints: accuracy of correction should be such that errors are negligible compared to the required photometric accuracy. Determining the non-linearity requires exposures with constant illumination over integration time. The calibration lamp flux should be stable enough for this.

Frequency/timing: daytime, monthly.

Feasibility: standard reduction: well-established correction schemes exist for the standard read-out scheme of IR detectors (reset-read-read mode).

6.4 Cold and hot pixels

Objective: create bad pixel mask

Accuracy/constraints: important for pipeline operations to fold into confidence map associated with each frame. Cold/dead pixels can be determined from instrumental flats with linear increasing/decreasing illumination and hot pixels from short darks.

Frequency/timing: to be determined during lab tests / commissioning.

Feasibility: standard reduction

6.5 Dark current

Objective: create dark current rate estimates for each pixel. This includes the zero-level offset ('bias'). This is a health-check

Accuracy/constraints: Since the dark counts will depend on the exposure time, the DIT of the dark observations and science observations must match.

Frequency/timing: daytime, daily health check instrument

Feasibility: standard reduction.

6.6 Flatfielding

Objective: create a map of pixel to pixel sensitivity variations per detector. The map is normalized to the average of all detectors to account for overall gain differences between detectors.

Accuracy/constraints: Maps are created by exposures with the internal calibration lamp (to determine pixel-to-pixel gains) and from twilight sky fields (to account for global non-uniformity of the internal flatfields; see 6.7).

Frequency/timing: daytime and twilight, daily

Feasibility: standard reduction

6.7 Illumination pattern

Objective: Correct for large-scale illumination variations over the detector plane. The gain variation over all detectors is characterized by the twilight and sky flatfields under the assumption of an ideal flat illumination over the field of view. In practice this ideal flat illumination can be affected by stray light (sky concentration). The data needs to be corrected for this. Based on the outcome more elaborate calibration schemes can be developed.

Accuracy/constraints: $\leq 1\%$ accuracy over focal plane. Shall be determined by observations of photometric standards dithered across detectors. 2MASS photometry can give a useful first estimate.

Frequency/timing: first estimate during commissioning and checks once per month and after each major setup change.

Feasibility: standard reduction

6.8 Persistence

Persistence – arising from residual charge in ‘charge traps’ – is a severe problem for current IR detectors (such as the HAWAII-2RG), particularly with bright or saturated sources. It can have a measurable impact for hours afterwards. The cause of persistence is not well-understood. The strength of persistence varies between IR detectors and not always repeatable.

Objective: Correct for persistence of counts from previous exposures.

Accuracy/constraints: the cause of persistence and how to mitigate the effect by special read-schemes shall be investigated. Correction schemes shall be based on these results.

Frequency/timing: correction potentially for every detector read-out.

Feasibility: requires further investigation

6.9 Sky subtraction

Objective: subtract the sky background image for on-sky images as constructed from observations taken close in time to the science image. For fields without large-scale objects the sky frame can be constructed by filtering out the objects from dithered observations. For observations which includes large scale objects the sky background is estimated from blank sky observations taken for this purpose.

Accuracy/constraints: Sky shall not vary significantly over the timespan of frames from which sky is created. The timespan is passband dependent as sky variations tend to be different for different wavebands. The optimal timespan shall be determined empirically during commissioning. Value of process parameters (minimum, maximum number of frames; single or iterative approach to mask-out astronomical sources from sky estimate), of generic subtraction method for sky subtraction shall be based on simulated data and verified during commissioning.

Frequency/timing: for every science exposure and each photometric and astrometric on-sky calibration frame

Feasibility: standard reduction

6.10 Photometric calibration

Objective: determine the gain of the combined system of atmosphere+telescope+instrument (i.e., zeropoints and atmospheric extinction) relative to an instrumental photometric system.

Accuracy and constraints: the standard requirement for the photometric calibration of the broad-band filters is to achieve an accuracy of better than 3% on the photometric scale in ‘instrumental magnitudes’ as assigned to the units of the resultant output image of the pipeline.

The colour transformation terms of instrumental to a standard system should be better than 20% on the photometric scale for photometric nights.

It shall be determined if standards with high enough accuracy exist at magnitudes fainter than the bright cut-off for the E-ELT+MAORY+MICADO observations by the time of operations. If not, the calibration plan shall contain an observing campaign to establish secondary standards. It shall be determined from simulations if the flux measurement of standards requires PSF fitting instead of aperture photometry to meet the requirement on photometric accuracy. The feasibility of PSF reconstruction shall be investigated to increase the highest possible photometric accuracy.

Frequency/timing: Deep atmospheric absorption bands (mainly due to water vapor) within filter passbands can have a significant impact on the effective transmission of atmosphere+filter observational system. The amplitude and timescales of atmospheric variations is site-dependent and shall be determined through a standard star observation over the full range of atmospheric conditions. This shall establish the requirement on allowed difference in time and atmospheric conditions compared to the observations to be calibrated. These observations should also establish the accuracy of colour terms for calibration from instrumental to a standard photometric system and correction of atmospheric extinction as a function of observing conditions. Many science cases require multi-epoch observations of the same field over a time span of typically months to years. In this case the photometric calibration to photometric standards on each night is desirable but not mandatory. Photometric calibration to the natural system can be done following relative calibration between epochs.

Feasibility: standard reduction except for the potential PSF fitting of standard star observations.

6.11 Astrometric calibration

The very high requirements on astrometry from several primary science drivers pose strong requirements on the astrometric calibration of MICADO. Therefore an overview is given first.

The aim of the astrometric calibration is to accurately determine the projection matrix for converting pixel positions to sky positions. Three types of astrometric calibration can be defined in general.

- **Local relative astrometry**: against internal reference source(s) within the FoV, i.e., observed simultaneously. The reference sources in this type of astrometric calibration are secondary standards and are instrument specific extrapolations from a standard astrometric reference catalog. The internal reference sources provide a limited accuracy position with respect to sky coordinates, but provide a high accuracy description of the instrumental profile, high positional accuracy within FoV. This is the type of astrometry required by the science drivers of MICADO up to an accuracy of 50microarcsec over the FoV, i.e., $1/60^{\text{th}}$ of a pixel in the Primary Arm.
- **Global relative astrometry**: against a set of reference points which cover a significantly larger sky area than FoV, i.e., not all references sources observed simultaneously. Combine multiple overlapping pointings to one astrometric unit. Multiple pointings can be from different epoch (5.15.4 and 5.15.5). This leads to same requirements as local relative astrometry. It requires multiple pointings with

considerable (>20%) overlap and use of internal reference sources/overlap sources. The current science drivers of MICADO do not require global relative astrometry.

- **Absolute astrometry:** against a global set of astrometric reference sources. The global set of astrometric reference sources must be external, for example the 2MASS catalog and meet certain requirements. The current science drivers of MICADO do not require absolute astrometry.

Four astrometric coordinate transformations were identified which form the framework for the astrometric calibration operations.

1. Mapping of position of detectors in focal plane
2. Mapping of geometric distortions
3. Intra-epoch astrometric calibration
4. Inter-epoch astrometric calibration

The four transformations are schematically shown in Figure 3.

A detailed assessment of the sources affecting the astrometric calibration and their error budgets is given in the Astrometry report in the Appendix. In the following we describe the astrometric calibration requirements for science observations which require maximum astrometric accuracy.

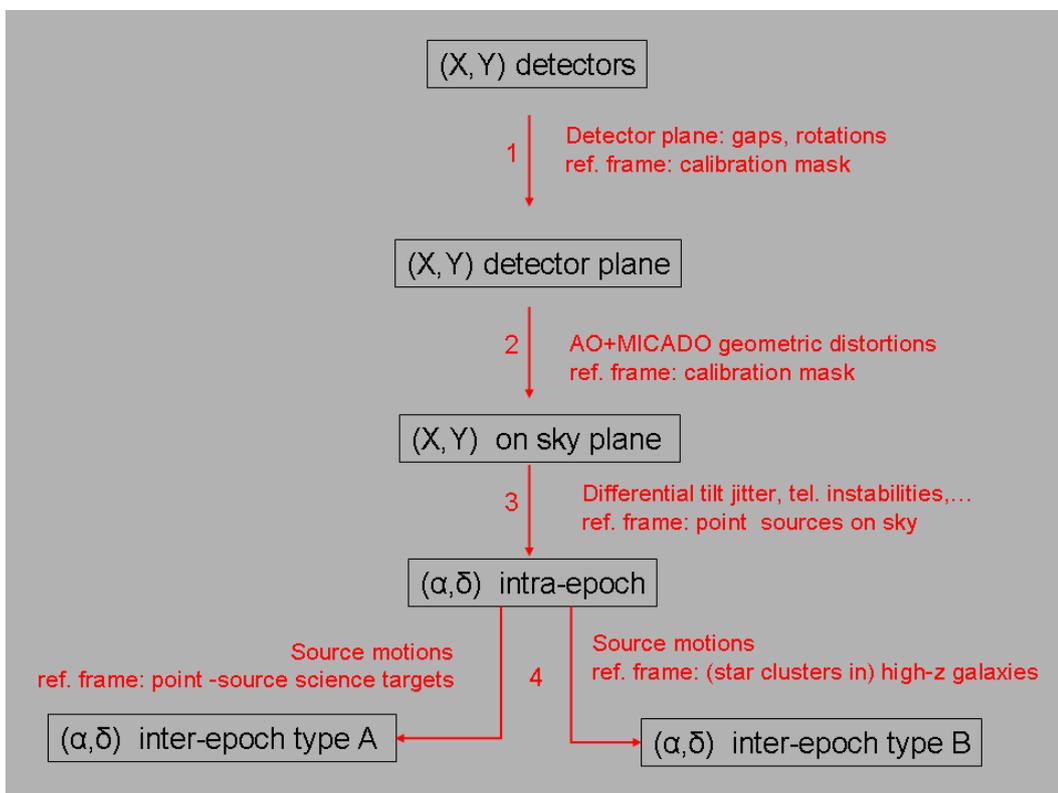


Figure 3: the high-precision astrometric calibration of MICADO requires 4 coordinate transformations (red arrows) between 5 coordinate systems. For each transformation the physical effect requiring the coordinate transformation is given together with the spatial/astrometric reference frame used.

6.11.1 Position of Detectors in Focal Plane

Objective: map the position of each pixel in each detector with respect to a perfect pixel plane. This means to map for each detector its shift, rotation and scale with respect to a perfect pixel plane. Dithered observations with the calibration mask will yield a set of 6 parameters per detector. Internal pixel mask inconsistencies are not measured.

Accuracy/constraints: map the relative pixel position geometry to precision of $\sim 2e-7$ over focal plane. The number of holes ('sources') in the calibration mask shall exceed 50 and should preferably be 100. The positional accuracy of holes needs to be $4e-8m$ relative to each other. The size of the holes needs to be $\leq 30e-6m$ to meet the diffraction limit. The holes pattern on the calibration mask should not to be a regular pattern but a Poissonian distribution of holes to allow absolute positioning of detectors relative to each other. The dependency of the calibration mask properties on temperature variations must be measured from lab experiments. The movement / positioning accuracy of the calibration mask is irrelevant for the calibration process. MAORY (or other AO system) needs to be in a locked state (neutral correction) because the calibration mask is in front of the AO system.

Frequency/timing: daytime, monthly initially, and also after each mechanical intervention to the camera

Feasibility: expected to be feasible. The progenitor (H4RG-10) of the baseline detectors (H4RG-15) are designed for space astrometry mission. The H4RG-15 is expected to meet the same requirements (priv. comm. Richard Blank of Astronomy & Civil Space, Teledyne Imaging Sensors). Positional pixel stability of the H4RG-10 is better than 10 nm. Each $4K \times 4K$ detector is made with a single pixel mask.

6.11.2 MICADO + MAORY Geometric Distortions

Objective: map the instrumental distortions induced by MAORY and MICADO. MICADO rotates with respects to MAORY. Therefore distortions from MAORY and MICADO need to be disentangled.

Accuracy/constraints: 30microarcsec accuracy of distortion model. Needs to be verified how/if the distortion is a function of time, telescope orientation, rotator angle, ambient temperature, atmospheric pressure and humidity. The distortions will be measured by imaging hole positions with very precise spacing. The requirements on the calibration mask are given in 6.12.1. The calibration procedure must use the MICADO rotator to disentangle MAORY from MICADO distortions. MAORY needs to be in a standard neutral state (e.g. DMs flat, and/or loops closed on internal reference fibre). The geometric corrections shall be cross-checked via on-sky observations. Distortions must be constant in time to enough accuracy: at least for 30min to allow shift-and-add procedure in T1 determination (see 6.12.4). Use MICADO all configurations to determine separate distortions per configuration setting.

Frequency/timing: to be determined after establishing which factors affect the distortions.

Feasibility: deemed feasible. See section 4.2 of the report in the Appendix for an assessment study. The MAORY distortion is expected to be 0.0018% over FoV (RD7) and for MICADO a few % over the FoV (RD5). Both distortions are expected to be constant in time to enough accuracy.

Note: Both 5.15.1 and 5.15.2 need to be derived simultaneously.

6.11.3 Source Positions

Objective: determine the positions of science targets and reference sources in pixel space. Find MAORY distortion geometry for non-neutral corrections in closed loop state.

Accuracy/Constraints: 5microarcsec accuracy at SNR of 10. Positions will be measured using PSF fitting. This requires determination of PSF model to 10^{-3} . Determination of non-neutral MAORY distortion is for the global FoV and is expected to be modelled with polynomial functions to order at least 3. Such a full polynomial description requires 20 parameters, yielding a minimum of 5×10 sources per FoV, preferably 100. Photometric observing conditions are required. The procedure is to shift-add single exposures of T0 (order few second) totalling an exposure time T1. The relative rotation between adjacent detectors shall be $\leq 1/500$ to allow shift-adding across detector gaps. Dither sizes shall be small enough to make non-linear distortions and effects of anisoplanatism negligible in accuracy of shift-adding T0 exposures. The T1 exposures need to be forward-regridded using the distortions as established in section 6.12.2. The T1 regrid has a total exposure time T2. The astrometric difference between sets of T2 added exposures should be a continuous polynomial. T2 shall be long enough that tip-tilt jitter can be integrated out over time, (see Appendix). T1 must be short enough that T0 exposures can be shift-added: i.e., non-linear effects are negligible over T1, such as AO distortions due to guide star measurement errors. Combining the T1 exposures requires regridding due to telescope instabilities. Time scales of instabilities to be determined empirically and from telescope design model.

Frequency/timing: can potentially be performed on every exposure, provided requirements are met.

Feasibility: deemed feasible. Astrometric calibrations which do not involve detailed PSF fitting commonly achieve positional accuracies of $1/10^{\text{th}}$ of a pixel. Inclusion of dedicated PSF fitting should improve this by an order of magnitude (see Section 4 of Astrometry report in Appendix for inventory of contributing effects and assessment of their error budgets).

Note: Instrumentally calibrated images are raw images with calibration header information, not regridded images. Regridding is required only as a disposable step in the derivation of the total astrometric calibration parameter set.

6.11.4 Intra-epoch astrometry

Objective: convert pixel-space positions to sky coordinates for sources in instrumentally calibrated images using reference sources in the FoV. Within time span of exposures the motion of science targets and reference sources is undetectable.

Accuracy/Constraints: 10 microarcsec accuracy on sky coordinates relative to ensemble of reference sources. Any set of instrumentally calibrated images can be used. . See 6.12.3 for the basic single epoch astrometric calibration requirements. Distortions within epochs are expected to be low-order polynomials. Astrometric reference sources of any kind can be used.

Frequency/timing: N/A

Feasibility: feasible (see 6.12.3).

6.11.5 Inter-epoch astrometry

Objective: Astrometrically calibrate observations at different epochs. Motion of science targets and possibly reference sources is detectable. Two types of astrometric reference systems are foreseen.

- TYPE A: construct astrometric reference system from science objects (e.g., assume random stellar motions in stellar system or radial linear expansion of science objects).
- TYPE B: high-z galaxies (and their star clusters) in observations are available as astrometric reference system.

Accuracy/Constraints: 10 microarcsec accuracy on sky coordinates relative to ensemble of reference sources. Any set of instrumentally calibrated images can be used. See sections 6.12.3 and 6.12.4 for the basic single epoch astrometric calibration. Distortions between epochs are assumed to be low-order polynomials.

Frequency/timing: any N/A

Feasibility: feasible (see section 6.12.3). When using high-z galaxies as astrometric reference points, integration times up to about 10 hours can be necessary (see Astrometry report in Appendix).

6.11.6 Monitoring astrometric stability

Objective: observe astrometric standard fields as part of the calibration plan. They are the astrometric analogous of photometric standard fields. This monitors the end-to-end relative astrometric accuracy of the E-ELT+MAORY+MICADO system.

Accuracy/Constraints: Definition of calibration fields with high density, non-crowded source population in all filters. The requirements on number density of the sources are as for the calibration mask. Sources should be high-z galaxies to have intrinsically constant astrometry.

Frequency/timing: to be determined from experience during commissioning

Feasibility: feasible.

6.12 Regridding and coaddition

Objective: resample images to a single uniform projection based on their astrometric calibrations and coadd them.

Accuracy/Constraints: <10microarcsec precision.

Frequency/timing: potentially any image for which astrometric solution has been derived.

Feasibility: feasible. The required mathematical resampling precision can be obtained by currently implemented approaches (e.g., Swarp) with straightforward adaptations.

7 REQUIREMENTS DATA REDUCTION SPECTROSCOPY

The E-ELT+ MAORY system give MICADO's spectroscopic capability immensely faint limiting magnitudes in combination with high spatial resolution.. In contrast to the astrometrically challenging image data reduction, MICADO's spectroscopy uses long-slits which are a well-established set-up in current instruments. Thus, the data reduction of MICADO's long-slit spectroscopy can be based to detail on reduction procedures for current day near-infrared spectrographs such as ISAAC and especially NACO. Therefore the top level requirements are only briefly summarized for spectroscopy

The data reduction for long-slit spectroscopy must be able to perform:

Sky-subtraction: The sky shall be determined from standard nodding or dedicated off-set sky positions and subsequently subtracted

Bad pixel masking: Requirements are those described in section 6.4.

Flat fielding: Generate a flat field by combining spatial information from a set of twilight sky exposures with spectral information from a set of calibration lamp exposures.

Combine, rectify and extract spectra: shift and combine 2D spectra, correct for spatial slit curvature before extracting 1D or 2D spectra.

Wavelength calibration: produce a wavelength map (wavelength for every detector pixel) from an arc frame, or from a night sky exposure with sufficient signal in the night sky OH emission lines. The pipeline should also be able to shift an existing wavelength map by a small amount to match a night sky exposure. The bootstrapping ability is especially important for the K band, where the night sky lines cover only a small fraction of the observing band.

Telluric line correction: correct science spectra for atmospheric emission-lines by scaling and subtracting observed telluric standard spectra. Sky conditions need to be similar for science target and telluric standard.

Flux calibration: flux-calibrate science spectra using spectral standards. Sky conditions need to be similar for science target and telluric standard.

8 DEVELOPMENT PLAN AND RISK ANALYSIS

Development of MICADO imaging data reduction pipeline

Partner NOVA includes its optical wide-field imaging expertise center OmegaCEN. The personnel has decades of expertise in this area. OmegaCEN has designed, developed and implemented the pipeline for ESO's OmegaCAM at the VST. This pipeline has been in full operation since December 2006 and thanks to its generality been applied to a dozen other optical imagers. Stand-alone version of this pipeline has been delivered to ESO. The optimal development track depends on the eventual data reduction and processing framework. The current baseline assumption is:

- Extending existing optical pipelines built by OmegaCEN to near-infrared imaging data reduction: many commonalities with optical data reduction in which case existing

pipelines in Astro-WISE can serve as basis. For IR specific pipeline aspects, the MICADO image pipeline will be based on near-infrared image reduction pipelines such as ISAAC at the VLT and VIRCAM-VISTA. As described in previous sections, many steps in the data reduction are very similar to those of other multi-detector near-infrared imagers.

- Extending astrometric pipeline of Astro-WISE to reduction of AO for extremely high astrometric accuracy. Many steps in the astrometric calibration foreseen for MICADO are covered by existing astrometric calibration procedures, also in Astro-WISE. The following main steps in the astrometric calibration for MICADO are not covered by astrometric calibration procedures in Astro-WISE:
 - Mapping of focal plane and geometric distortions with an astrometric calibration mask: feasible as discussed in Section 6.11.
 - PSF fitting: not incorporated in Astro-WISE: can be based on or make use of existing expertise: e.g., PSFex, starfinder, DAOPHOT (see RD6).
 - INTER-EPOCH astrometric calibration TYPE A is not available in common astrometric pipelines.

Development of MICADO spectral data reduction pipeline

MICADO IR spectral pipeline shares many commonalities with existing IR spectral data reduction pipelines such as those for ISAAC and NACO. Since spectroscopic data volumes are an order of magnitude less than for imaging: it needs to be determined during PDR if it is more user-friendly and efficient to incorporate the spectral data reduction in the same infrastructure as the imaging data reduction or in an independent infrastructure.

Development of MICADO data reduction infrastructure

The development of the MICADO imaging data reduction infrastructure can be based on the experience gained from developing the Astro-WISE infrastructure developed by partner NOVA-OmegaCEN (Valentijn et al 2008). The system is scalable by design: being applied to data reduction of dozen optical imagers operating data in the 10s TB regime. The infrastructure is also being applied to LOFAR which has Petabyte data volumes. As used for Astro-WISE the approach of Component Based software engineering (CBSE) will be used for MICADO. This will also facilitate the interface to existing data reduction infrastructures if needed.

Simulated data reduction

E-ELT with its multi-conjugate adaptive optics brings especially the image data reduction to an unprecedented regime. Detailed simulated data and data reduction will therefore be crucial to test various approaches in this new regime. From pre-phase B on data simulations will be made and data reduction simulations will be performed.

9 APPENDIX: ASTROMETRY REPORT TRIPPE ETAL.

Attached to this document is a paper entitled “High Accuracy Astrometry with MICADO at the European Extremely Large Telescope” by Trippe, Davies, Eisenhauer, Förster Schreiber, Fritz and Genzel. This work was carried out as part of the MICADO astrometry study, and the resulting paper has been submitted to MNRAS.

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High Precision Astrometry with MICADO at the European Extremely Large Telescope

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ABSTRACT

In this article we identify and discuss various statistical and systematic effects influencing the astrometric accuracy achievable with MICADO, the near-infrared imaging camera proposed for the 42-metre European Extremely Large Telescope (E-ELT). These effects are instrumental (e.g. geometric distortion), atmospheric (e.g. chromatic differential refraction), and astronomical (reference source selection). We find that there are several phenomena having impact on $\sim 100\mu\text{as}$ scales, meaning they can be substantially larger than the theoretical statistical astrometric accuracy of an optical/NIR 42m-telescope. Depending on type, these effects need to be controlled via dedicated instrumental design properties or via dedicated calibration procedures. We conclude that if this is done properly, astrometric accuracies of $40\mu\text{as}$ or better – with $40\mu\text{as/yr}$ in proper motions corresponding to ≈ 20 km/s at 100 kpc distance – can be achieved in one epoch of actual observations.

Key words: Telescope — Astrometry — Instrumentation: high angular resolution — Techniques: high angular resolution

1 INTRODUCTION

The future optical/near-infrared European Extremely Large Telescope (E-ELT; see, e.g., Gilmozzi & Spyromilio (2008)), which is designed with a 42-metre aperture, will offer a substantial improvement in angular resolution compared to existing facilities. At wavelengths $\lambda = 2\mu\text{m}$, diffraction-limited resolutions of $\Theta \approx 10\text{mas}$ will be achieved. In terms of angular resolution in the near infrared, the E-ELT will outperform existing 8–10m-class telescopes like the VLT or Keck by factors of ≈ 4 –5 and the future James Webb Space Telescope (JWST) by factors of ≈ 7 . This increase in angular resolution should translate into a corresponding improvement in astrometric accuracy.

In order to exploit the E-ELT's resolution, a German-Dutch-Italian-French consortium¹ proposed the Multi-AO Imaging Camera for Deep Observations (MICADO) in February 2008. As the spatial resolution of any ground-based observatory is initially limited by the atmospheric seeing, MICADO will be equipped with a multi-conjugate adaptive optics (MCAO) system for achieving the diffraction limit of the 42m-telescope. This system uses three natural and six laser guide stars for correcting the atmospheric turbu-

lence in a wide ($> 2'$) field of view (Diolaiti et al. (2008)). Images will be recorded by an array of 4×4 near-infrared (NIR) HAWAII-4RG detectors with 4096×4096 pixels each, covering a FOV of $53'$. The instrument is sensitive to the wavelength range $0.8 - 2.5\mu\text{m}$, thus covering the I, Y, J, H, K bands. For astrometric experiments the use of the data analysis software Astro-WISE (Valentijn et al. (2007)) is foreseen.

In order to achieve its science goals (see Sect. 2 for details), MICADO needs to reach a stable (time scales of years) astrometric accuracy of approximately $50\mu\text{as}$. At present 8–10m class telescopes, accuracies of $\approx 0.5\%$ of a resolution element can be reached regularly (e.g. Fritz et al. (2009)). Therefore from simple scaling of results our goal *a priori* appears reasonable. However, at levels of the order of $100\mu\text{as}$ there are several sources of statistical and systematic errors which need to be taken into account carefully. In this article we discuss those effects and analyse strategies to bypass them. We conclude that reaching an astrometric accuracy of better than $50\mu\text{as}$ is highly challenging in terms of instrument design and data calibration but feasible.

MICADO's astrometric performance should be of the same magnitude as that of the future astrometry space mission GAIA (e.g. Jordan (2008)). GAIA will achieve accuracies better than $\approx 50\mu\text{as}$ only for bright ($V < 15.5$) targets and only at the end of its mission. MICADO is expected to achieve this accuracy for targets with $K_{AB} < 26$. Other space missions like SIM PlanetQuest (e.g. Edberg et al. (2007)) or JASMINE (e.g. Gouda et al. (2007))

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¹ The MICADO collaboration includes: MPE Garching, Germany; USM Munich, Germany; MPIA Heidelberg, Germany; NOVA (a collaboration of the universities of Leiden, Groningen, and ASTRON Dwingeloo), The Netherlands; OAPD Padova (INAF), Italy; LESIA Paris, France

also aim specifically at bright targets in order to reach accuracies of $\approx 10\mu\text{as}$ (at best).

For illustration purposes, Fig. 1 shows simulated observations of the nuclear star cluster of the Milky Way using both present day 8-10m class telescopes and E-ELT/MICADO. Physical parameters of the star cluster (stellar density profile, luminosity function) are taken from Genzel et al. (2003). We discuss technical details of our simulations in Sect. 4.1. These maps demonstrate the impressive progress to be expected with MICADO.

Although this study is set up for the specific case of MICADO, most of its results are valid in general and therefore of interest beyond the E-ELT community.

This paper is organised as follows. In Section 2, we discuss the science cases identified for MICADO. In Section 3, we review the concepts and techniques of accurate astrometry. In Section 4, we identify and analyse sources of systematic errors one by one and describe methods for minimizing those errors. We provide a summary of our results and an overall error budget in Section 5 and present our conclusions in Section 6.

2 SCIENCE CASES

As part of the instrument design study, the MICADO collaboration has identified and analysed (Renzini et al. (2008), and references therein) several science cases for which the high astrometric accuracy of E-ELT/MICADO is crucial and promises major discoveries. We discuss them in the following one-by-one.

2.1 Galactic Centers

Located at a distance of ≈ 8 kpc, the nuclear region of the Milky Way is the closest galactic nucleus, hosting the supermassive black hole ($M_{\bullet} \approx 4 \times 10^6 M_{\odot}$) Sgr A* (e.g. Gillessen et al. (2009)). It is therefore a unique laboratory for exploring the regime of strong gravity, accretion onto black holes, and the co-evolution of dense star clusters and active galactic nuclei.

Present-day NIR instrumentation, e.g. VLT/NACO, provides astrometric accuracies down to ≈ 0.3 mas and angular resolutions down to ≈ 50 mas (e.g. Fritz et al. (2009)). This allowed to identify several stars on Keplerian orbits around Sgr A* with orbital periods down to ≈ 15 years and pericenter distances as small as ≈ 100 AU (≈ 12 mas on sky; e.g. Gillessen et al. (2009)). It made possible to study in detail the kinematics and the composition of the nuclear star cluster in the gravitational potential of the central black hole. With E-ELT/MICADO one can expect to achieve sensitivities that are more than five magnitudes fainter than for VLT/NACO. Angular resolutions and astrometric accuracies should also improve by factors of about five, meaning that proper motions of order $10\mu\text{as/yr}$ (400 m/s) can be detected within few years of observations. Such instrumental performance is necessary in order to address several new questions (e.g. Gillessen et al. (2009)):

- Identification of stars on closeby Keplerian orbits with periods of few years.
- Measuring the *prograde* relativistic orbit precession and testing other effects of general relativity.
- Probing possible *retrograde* orbit precession due to an extended mass component built from compact stellar remnants.
- Analyzing the various separate kinematic structures of the nuclear star cluster, and searching for new ones.
- Quantifying the binary star fraction in the nuclear cluster.

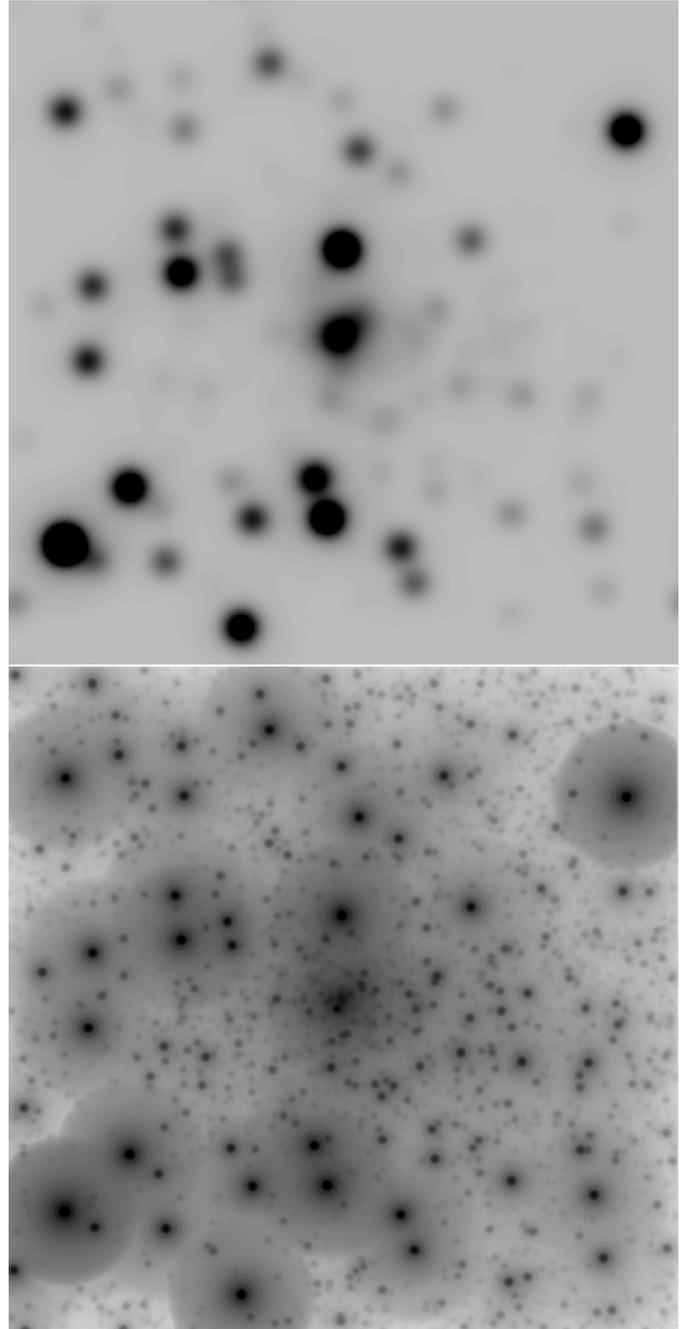


Figure 1. An illustration of the expected performance of the E-ELT/MICADO system. These simulated maps show the central $1'' \times 1''$ (i.e. $8000\text{AU} \times 8000\text{AU}$) of the nuclear star cluster of the Milky Way at $2.2\mu\text{m}$. *Top panel:* The target region as observed with present day 8-10m class telescopes. The diffraction-limited resolution is $\approx 50\text{mas}$. For comparison with actual observations, see, e.g., Genzel et al. (2003), Ghez et al. (2005). *Bottom panel:* The same field as seen by MICADO. The angular resolution is $\approx 10\text{mas}$. The improvement in detail and depth is obvious.

With MICADO, this type of analysis can be extended to other nearby galaxies. One obvious example is the core of M31 which hosts a $M_{\bullet} \approx 1.4 \times 10^8 M_{\odot}$ black hole. Similar to the case of the Milky Way, M31's nucleus shows several distinct stellar populations: a triple nucleus and two nested star disks around the central black hole have been identified (Bender et al. (2005)). Although M31 is

more distant from earth by a factor ≈ 100 compared to the Galactic center, the larger mass of its black hole (by a factor of ≈ 35) causes stellar proper motions of about 6% compared to those in the nuclear cluster of the Milky Way. Therefore kinematic analyses analogous to the Galactic center experiment will require capabilities as predicted for E-ELT/MICADO.

Another example is Centaurus A which hosts a $M_{\bullet} \approx 5 \times 10^7$ black hole (Neumayer et al. (2007)). Given its distance of ≈ 5 Mpc, proper motions of $\approx 10 \mu\text{as/yr}$ correspond to ≈ 200 km/s. Thus measuring the motions of stars in the vicinity of the black hole is possible. Other galaxies might be interesting targets as well.

2.2 Intermediate Mass Black Holes

The expected high astrometric accuracy of MICADO opens a new window in the search for and analysis of intermediate mass black holes (IMBH), objects with masses of few thousand solar masses. In the last years, the detection of those objects in the Arches cluster (Portegies Zwart et al. (2006)), the star association GCIRS13 in the Galactic center (e.g. Maillard et al. (2004)), ω Cen (Noyola et al. (2008)), and other locations has been claimed. Most of these analyses are based on radial velocity dispersion profiles. This introduces systematic ambiguities as anisotropic velocity dispersions can mimic the presence of central point masses; this one can see for example in the anisotropy term in the Jeans equation. Therefore reliable (non)detections of IMBHs require measurements of stellar proper motions (e.g. Anderson & van der Marel (2009)). Typical velocity dispersions σ_* of star clusters are of the order 10 km/s. This corresponds to $\approx 50 \mu\text{as/yr}$ at a distance of 40 kpc, meaning that for most of the Galactic star clusters a proper motion analysis is feasible with MICADO only. This allows

- constraining black hole masses in Galactic star clusters within few years,
- probing the low-mass end of the $M_{\bullet} - \sigma_*$ relation,
- testing the dynamical evolution of star clusters.

2.3 Globular Clusters

As discussed above, the astrometric accuracy of MICADO should allow measuring stellar proper motions of few km/s for most Galactic globular clusters. Additionally, direct measurements of cluster parallaxes become possible: for a distance of 40 kpc, the full parallax displacement is $50 \mu\text{as}$, corresponding to MICADO's predicted astrometric accuracy. This allows addressing several topics:

- The spatial distribution of globular clusters.
- Cluster proper motions and their orbits around the Milky Way (e.g. Bedin et al. (2003; 2006)).
- Internal cluster kinematics, including rotation.
- Separating cluster members from field stars, thus making analyses of cluster star populations more reliable (see also Anderson et al. (2006)).

2.4 Dark Matter In Dwarf Spheroidal Galaxies

Cold dark matter models predict high mass densities and cuspy density profiles for the central regions of galaxy halos. In contrast, warm dark matter models predict substantially lower central densities and constant density cores at small radii. The dwarf spheroidal satellite galaxies of the Milky Way provide a unique laboratory to test those models. Their proximity of ≈ 100 kpc makes it possible

to resolve individual stars and analyze their dynamics in the gravitational potentials of their galaxies. Present day studies are usually based on line-of-sight velocity dispersion profiles. However, there are degeneracies of velocity dispersion anisotropies with mass density profiles. Therefore any conclusive analysis requires measuring all three components of the velocity vectors of the tracer stars (e.g. Strigari et al. (2007)).

This type of studies requires accurate proper motion measurements with uncertainties of few km/s or better. MICADO will be able to provide accuracies of ≈ 5 km/s within few years of observations for targets about 100 kpc away. This makes MICADO a decisive tool for testing the validity of present-day dark matter models.

3 THE ASTROMETRY PROBLEM

Throughout this paper, we use the term ‘‘astrometry’’ the following way. We discuss time-resolved *relative* positions, meaning the positions of a science target with respect to a set of reference sources. Science target and reference sources are located in the field of view (FOV) of the camera, i.e. they are recorded simultaneously in the same science image.

Intra-epoch measurements cover timelines that are so short that intrinsic motions of science targets or reference sources cannot be detected. This can be a set of images taken within the same night or a few adjacent nights. Any variations in measured positions are due to measurement errors and can be used to determine the position accuracies. All information obtained from this data set (i.e. images, coordinates, ...) can be combined, e.g. for improving the signal-to-noise ratio (SNR).

Inter-epoch measurements cover timelines sufficient for detecting source motions. Those data cannot be combined in a straight forward manner. Intrinsic source motions and measurement errors interfere and need to be disentangled, usually meaning that calculating errors requires additional information. Given MICADO's proper motion accuracies of $\approx 50 \mu\text{as/yr}$ or better (depending on the duration of the experiments), corresponding to ≈ 10 km/s at 40 kpc distance, intrinsic motion of Galactic stars is detected easily. Therefore it will be necessary to use extragalactic sources, including high redshift objects like QSOs, as references for some science cases.

If a set of science images is at hand, the general analysis recipe is as follows.

Step 1. From each image n , one extracts the detector positions $\{\mathbf{X}^n\}$ of all sources of interest (science targets and reference sources). Detector positions need to be measured with high accuracies of order few milli-pixels (mpix). Existing centroiding, source profile fitting, and point spread function (PSF) correlation algorithms provide such accuracies (e.g. Diolaiti et al. (2000); Berry & Burnell (2000); Trippe (2008)).

Step 2. For each image n , the detector positions $\{\mathbf{X}^n\}$ need to be converted into global astrometric coordinates $\{\mathbf{x}^n\}$. The reference frame can be the detector coordinates of a selected zero-point image or any more general astrometric coordinate system. Using the detector positions of the reference sources $\{\mathbf{X}_{\text{ref}}^n\}$ and their astrometric positions $\{\mathbf{x}_{\text{ref}}^n\}$, one calculates a transformation

$$T_n : \{\mathbf{X}_{\text{ref}}^n\} \longrightarrow \{\mathbf{x}_{\text{ref}}^n\}. \quad (1)$$

Obtaining the positions $\{\mathbf{x}_{\text{ref}}^n\}$ requires some prior knowledge on the reference sources. One powerful approach is cross-calibration with other datasets, maybe from other wavelength regimes. A nice example is given by Reid et al. (2007). They use precise VLBI astrom-

etry of SiO maser stars in order to define an astrometric reference frame in K-band images of the Galactic center.

If the reference sources are known to be not moving (e.g. extragalactic objects), one can set $\{\mathbf{x}_{\text{ref}}^n\} = \{\mathbf{X}^0\}$; the index 0 indicates the selected zero-point in time. The same can be done if the reference source ensemble is (or is defined as being) at rest *in average*, i.e. $\langle\{\mathbf{x}_{\text{ref}}^0\}\rangle = \langle\{\mathbf{x}_{\text{ref}}^n\}\rangle$ (e.g. a sufficiently large set of stars in a star cluster). In this case however, one will lose information on a global motion (drift, rotation, contraction, etc.) of the combined system “science target + reference sources”. In any case, the transformation T_n is used to compute global astrometric coordinates for the science targets like

$$\{\mathbf{X}^n\} \longrightarrow T_n(\{\mathbf{X}^0\}) = \{\mathbf{x}^n\}. \quad (2)$$

Commonly, low-order (<5) 2-dimensional polynomial coordinate transformations

$$x' = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + \dots \quad (3)$$

$$y' = b_0 + b_1x + b_2y + b_3x^2 + b_4y^2 + b_5xy + \dots \quad (4)$$

are used. In case prior knowledge on the geometry of the required transformation is available, one can use models with smaller numbers of free parameters (e.g. Montenbruck & Pfleger (1989); Anderson et al. (2006); Trippe et al. (2008)).

The number of available reference sources governs the maximum order of coordinate transformations (see step 2). A 1st order polynomial transform with six parameters requires three reference sources, i.e. $2 \times 3 = 6$ coordinates. A 2nd-order transform (12 parameters) requires six reference sources, and so on. Throughout this paper we assume that *any two science images need to be connected via full astrometric transformations*. This means that we regard more simple methods of data combination like stacking, simple-shift-and-add etc. as non-astrometric and thus not usable for our purpose.

In order to judge the astrometric accuracy achievable with a system like MICADO, one has to distinguish statistical and systematic influences. The *statistical* measurement accuracy is given by

$$\sigma_L = \frac{\lambda}{\pi D} \frac{1}{SNR} = 284 \mu\text{as} \left(\frac{\lambda}{2.17 \mu\text{m}} \right) \left(\frac{5\text{m}}{D} \right) \left(\frac{100}{SNR} \right) \quad (5)$$

Here λ is the wavelength, D the telescope aperture, and SNR the signal-to-noise ratio (Lindgren (1978)). To give an example for the case of the E-ELT: with $\lambda = 2.2 \mu\text{m}$ (K-band), $D = 42\text{m}$, and $SNR = 100$, one obtains a statistical astrometric accuracy $\sigma_L = 34 \mu\text{as}$. From this we see that – in principle – astrometric accuracies of $\sim 10 \mu\text{as}$ can be obtained with the E-ELT. This means that any source of additional, especially *systematic* error needs to be compensated down to this level if one actually wants to fully exploit the E-ELT’s capabilities.

4 ERROR SOURCES

In total, we have identified ten effects that might have the potential to reduce the expected astrometric accuracy of MICADO substantially. We will discuss these “Terrible Ten” in the following subsections. The first three phenomena we analyze are instrumental, the next five are atmospheric, and the last two are astronomical.

Table 1. Strehl ratio estimates as obtained from preliminary simulations (Liske 2008; M. Kissler-Patig *priv. comm.*). The λ_{center} are the central wavelengths of the filters.

Band	I	J	H	K
$\lambda_{\text{center}} [\mu\text{m}]$	0.900	1.215	1.654	2.179
Strehl ratio [%]	2	18	35	53

4.1 Sampling and Pixel Scales

The detector position of a point source can be computed only if the source PSF is sufficiently sampled. If the pixel scale – expressed in angular units per pixel – is too large (undersampling), position information is lost because there is no unique mathematical description for the PSF profile anymore. Especially, a PSF can then be modelled by profiles with different centers of light (i.e. different detector positions). This effect is known as the *pixel phase error*. It can reach magnitudes of several tenths of a pixel, thus providing an important boundary condition for the instrument design. A detailed description of this phenomenon is provided by Anderson & King (2000).

In order to identify the critical pixel scale of MICADO, we first created artificial PSFs for each of the bands I, J, H, and K. We modelled each PSF $P(\mathbf{x})$ as a superposition of a 2-dimensional Airy function $A(\mathbf{x})$ and a 2D Moffat profile $M(\mathbf{x})$ like

$$P(\mathbf{x}) = aA(\mathbf{x}) + (1 - a)M(\mathbf{x}). \quad (6)$$

For each PSF, a stochastic optimization routine² adjusted the parameter $a \in [0, 1]$ such that the resulting PSF profile showed the proper Strehl ratio. We took Strehl ratio estimates for MICADO from preliminary E-ELT adaptive optics (AO) system simulations (Liske (2008); M. Kissler-Patig *priv. comm.*), the values are shown in Table 1.

For each of the four wavelength bands we examined pixel scales from 1 to 7 mas/pix in steps of 0.1 mas/pix. In each configuration, we placed the PSF at a random detector position and re-binned it to the corresponding pixel scale. After this, we fit the detector position with a 2D Gaussian light distribution. We iterated this procedure 250 times for each configuration. Pixel phase errors were the rms values of the distributions of the differences between true and measured positions.

The results of our analysis are shown in Fig. 2. For clarity, we restrict the diagram to the decisive pixel scale range from 2 to 4 mas/pix. For all wavelength bands the errors are smaller than $\approx 1 \mu\text{as}$ for pixel scales below 3 mas/pix. Given that the filter bands span a factor of 2.4 in wavelength and thus in λ/D , it might not be obvious why the critical pixel scales we find are that similar for all bands. This effect is caused by the differences in Strehl ratios: whereas the diffraction limited profile width decreases with decreasing wavelength, the Strehl ratio decreases, too, meaning a stronger atmospheric blur. In our case, the two effects roughly counterbalance each other; we actually find the smallest beam size for the H band PSF due to its combination of high SNR ($\approx 35\%$) and small diffraction limit ($\lambda/D \approx 8.1$ mas). This makes it possible to quote a single critical pixel scale for all bands. We therefore conclude that for *isolated* point sources pixel scales up to 3 mas/pix can

² Implemented in the MPE data processing software DPUSER developed by Thomas Ott; see <http://www.mpe.mpg.de/~ott/dpuser/history.html>

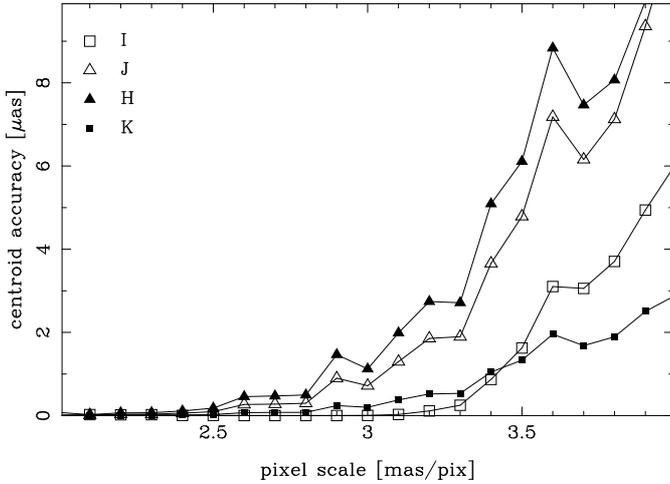


Figure 2. Pixel phase error vs. pixel scale for I, J, H, K bands for *isolated* sources. Model PSFs are superpositions of Airy and Moffat profiles with Strehl ratios as given in Table 1. For all bands, the errors are below $\approx 1\mu\text{as}$ for pixel scales smaller than 3 mas/pix.

be used for MICADO without introducing noticeable ($\approx 1\mu\text{as}$) pixel phase errors. Therefore we use this scale as the standard value for the MICADO design.

In case of *crowding*, the astrometric accuracy is additionally limited by source overlap. For this reason the MICADO design foresees a “small scale mode”. As the camera design is catoptric and all mirrors are fixed, we use a pick-up arm to image a small (6”, using one out of the 4×4 detectors) FOV with a reduced pixel scale of 1.5 mas/pix. The reduced scale helps to better separate close by sources and such reduce the misplacements of the source centroids.

In order to quantify this effect, we examined our two pixel scales, 1.5 mas/pix and 3 mas/pix, for each of the bands H and K, meaning four configurations in total. For each configuration we simulated a crowded star field by placing PSFs at random positions. We used the same model PSFs as for the case of isolated sources. We applied a luminosity profile corresponding to the K-band luminosity function of the Galactic bulge which is $d \log n / d \log S \approx -0.8$ (Zoccali et al. (2003); n is the source number, S is the flux). The dynamic range of the star sample was 10 magnitudes, i.e. a factor of 10,000 in flux. Source densities were ≈ 3000 stars per square arcsecond. We also used these routines (with modified parameters) to create the map shown in Fig. 1.

In each simulated map, we searched for stars and calculated their detector positions using the PSF fitting routine StarFinder (Diolaiti et al. (2000)). We derived median uncertainties vs. fluxes from the distributions of the differences between true and measured positions. As the absolute values of these errors are functions of several parameters like fluxes, luminosity profiles, and source densities, we converted our results into relative numbers, using the errors obtained for the 3 mas/pix scales as references.

The outcome of our analysis is presented in Fig. 3. Reducing the pixel scales from 3 mas/pix to 1.5 mas/pix improves the typical accuracies by factors of $\approx 2-3$. The effect is stronger in H than in K band; this indicates that images with smaller beam sizes profit more from a small pixel scale. For the brightest sources (more than ≈ 500 units) the differences between the pixel scales are (at least in K band) not very pronounced any more because (a) the number of very bright sources is small and (b) they outshine most of their neighbours. We can thus conclude that for *crowded* fields the use

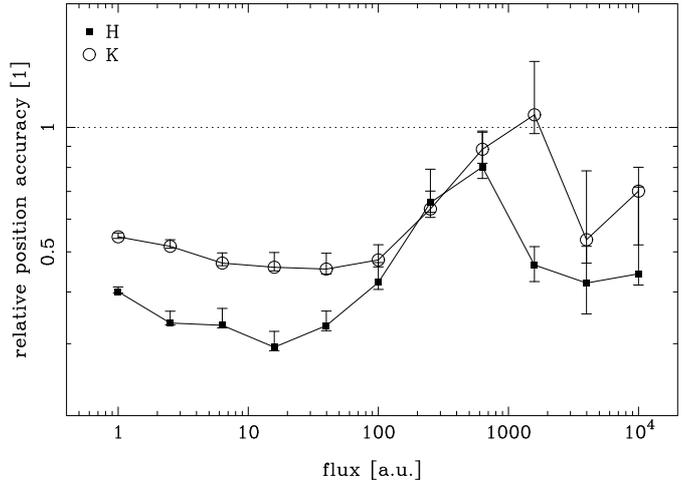


Figure 3. Relative median astrometric error vs. source flux for *crowded* sources, separately for H and K bands. The results for the pixel scales 1.5 mas/pix are given as fractions of the uncertainties found for 3 mas/pix. The horizontal dashed line indicates a ratio of 1. Error bars indicate the 68% uncertainty ranges. This analysis shows that reducing the pixel scale improves the typical accuracies by factors $\approx 2-3$.

of small pixel scales down to 1.5 mas/pix is indeed important for keeping a high level of astrometric accuracy.

Combining all results for isolated and crowded sources, we can conclude the following for the design and the operation of MICADO:

- For sufficiently *isolated* target sources, pixel scales smaller than or equal 3 mas/pix are free of noticeable ($\approx 1\mu\text{as}$) systematic uncertainties and suited for accurate astrometry. This scale is therefore going to be the standard pixel scale for MICADO.
- For the special case of *crowded* sources, smaller pixel scales down to 1 mas/pix can substantially (factors ≈ 2) reduce the astrometric errors introduced by source overlap. We therefore implemented a “small-scale mode” with a reduced pixel scale of 1.5 mas/pix into the MICADO design. This mode will be used to map very crowded regions.

4.2 Instrumental Distortion

The geometric distortion of an optical system can seriously limit its astrometric accuracy. In the following, we discuss *non-linear* distortions. *Linear* terms like shifts, rotations, scalings, or shear, are absorbed by 1st (or higher) order coordinate transforms (see Sect. 3). In case of non-linear distortion, the effective pixel scale is a function of detector position (e.g. Greason et al. (1994), and references therein).

For MICADO, the amount of distortion to be expected (meaning the difference between imaged and theoretical image positions) is of the order of few per cent. The largest numbers, about 5%, we find for the case of an Offner design for the camera. Although other optical designs might have the potential to reduce the amount of distortion down to $\approx 0.3\%$ (Dierickx (2008)), it cannot be neglected. Across a FOV of $1'$, a distortion of 0.3% corresponds to a misplacement of $0.18''$. For comparison: an astrometric accuracy of $\approx 50\mu\text{as}$ across the same FOV corresponds to a relative error of less than 10^{-6} . Such low uncertainties cannot be achieved by design. In order to reach the desired astrometric accuracies, the distortion must

be compensated by dedicated calibration schemes – regardless of its amplitude.

Another effect to be considered is imperfect fabrication of the detectors. The HAWAII-4RG detectors to be used for MICADO are designed with pixels sizes of $15\mu\text{m}$. The position accuracy of these pixels in the detector grid is $\approx 10\text{nm}$; for a pixel scale of 3 mas/pix this corresponds to an astrometric uncertainty of $\approx 2\mu\text{as}$ (Richard Blank³ *priv. comm*), i.e. is hardly noticeable. However, any distortion calibration scheme must provide the ability to catch potential inaccuracies of the detectors (see also Anderson (2002) for the case of HST/WFC).

Depending on the complexity of the distortion, two methods for its description are possible. *Analytic* descriptions make use of analytic parametrizations analogous to the coordinate transforms discussed in Sect. 3. Most commonly, 2D polynomials up to about 5th order are used as distortion models (e.g. Greason et al. (1994), and references therein). This approach allows covering all effects up to a selected order without prior knowledge on the geometry of the problem. The disadvantage of this method is the large number of model parameters to be calculated. A recent example for this approach is the analysis of the nuclear star cluster of the Milky Way by Ghez et al. (2008) and Lu et al. (2009). They use a polynomial model to correct the geometric distortion of the NIRC and NIRC2 cameras at the W.M. Keck Observatory.

If a physical model for the distortion is at hand, a corresponding model can be substantially simpler. A recent example for this approach is the analysis of the nuclear star cluster of the Milky Way by Trippe et al. (2008) and Gillessen et al. ((2009)). They use the 3rd-order model

$$\mathbf{r} = \mathbf{r}'(1 - \beta\mathbf{r}'^2) \quad (7)$$

with

$$\mathbf{r} = \mathbf{x} - \mathbf{x}_C \quad \text{and} \quad \mathbf{r}' = \mathbf{x}' - \mathbf{x}_C$$

(e.g. Jähne (2005))⁴ in order to correct the distortion of the imager NAOS/CONICA at the VLT. Here \mathbf{x} and \mathbf{x}' are the true and distorted image coordinates respectively, β is a parameter describing the strength of the grid curvature, and $\mathbf{x}_C \equiv (x_C, y_C)$ is the zero point of the distortion on the detector. This approach has the obvious advantage that it requires only three parameters (β, x_C, y_C) for a 3rd-order distortion model compared to 20 for the case of a full polynomial solution. The most important disadvantage is the need for accurate *a priori* knowledge of the distortion geometry.

In cases where analytic solutions are not feasible or not accurate enough, *empiric* descriptions might be used instead or in addition. This means that the information of interest is stored in look-up tables. Those tables usually have the dimensions of the detector(s) and give the amount of distortion (or correction) for each pixel (separately for x, y). This approach is necessary if significant high-frequency distortion – meaning that the spatial scales of the signal are small compared to the detector size – is present. For the specific case of MICADO, the gaps between the detectors will introduce discontinuities into any astrometric solution. This effect could be caught by using lookup tables.

In a detailed analysis of the WFC instrument aboard the Hubble Space Telescope, Anderson (2002) uses a combined “polyno-

mial model plus lookup-table” ansatz to model the distortion of the camera. He achieves residuals smaller than 0.01 pixels with this method. For the case of MICADO, this would mean residual errors below $\approx 30\mu\text{as}$ which is an amount acceptable for our purpose.

There are several methods to extract the distortion parameters. *On-sky methods* make use of dedicated observations of astronomical targets; usually, star clusters are used (e.g., Anderson (2002)). If the true (astrometric) coordinates of the sources in the target field are known, one can derive the distortion parameters by comparing true and observed (detector) positions. If the true source positions are not known, one can observe the target field many times with slightly different telescope pointings. In this case, one compares the pairwise distances of objects that are present in two or more images. Modulations in these distances as function of detector position are equivalent to (non-linear) distortion.

By construction, on-sky methods are sensitive to distortions introduced by the atmosphere. Given the high accuracies we seek, those methods are not practicable (at least stand-alone) for the case of MICADO but will be a secondary approach.

In-lab methods characterize the instrument with dedicated measurements in the laboratory or at the telescope. One example is the “north-south test” used for the spectro-imager SINFONI at the VLT (e.g. Abuter et al. (2006)). These methods use devices that illuminate the detectors with well-defined images or light patterns. Comparing the theoretical with the observed images allows for a description of the distortion.

Given that in our accuracy regime the atmosphere can severely limit the quality of on-sky calibration images (see Sect. 4.4 – 4.8 for details), we assume that we need to implement an internal calibration device into MICADO. For this purpose, we examined the use of a calibration mask located in the focal plane of the imager. Such a mask could be a regular pattern of holes in an intransparent material. However, the following calculation shows that this is actually challenging. With MICADO, we want to observe a FOV with extension $x = 1'$ with an accuracy $\delta x = 10\mu\text{as}$, meaning a relative accuracy of

$$\delta x/x \approx 1.7 \times 10^{-7} .$$

The width of MICADO’s focal plane will be approximately $l = 0.25\text{m}$. This scales to a positioning accuracy for any reference located in the focal plane of

$$\delta l = l \times \delta x/x \approx 4.2 \times 10^{-8}\text{m} .$$

As we see here, the positions of the holes in our calibration mask need be known with accuracies of about 40nm (the hole diameters would be $\approx 30\mu\text{m}$ corresponding to diffraction-limited point sources). This result does not mean that the fabrication process needs to be accurate at this level. Instead, it is sufficient to map the hole positions with accuracies of $\approx 40\text{nm}$ after the making of the mask. State-of-the-art photolithographic techniques provide the required production and mapping accuracies at standard temperatures ($\approx 300\text{K}$). With this approach, the calibration mask would be a transparent quartz plate with a chrome cover into which the holes are etched. Although such a system is thermally very stable, the impact of possible non-linear thermal deformation needs to be investigated via dedicated laboratory experiments (B. Lorenz⁵, D. Rose⁶ *priv. comm.*). We consider the calibration mask approach to be the

³ Teledyne Imaging Sensors, Camarillo, USA

⁴ See also the electronic manual of the public Gemini North Galactic Center Demonstration Science Data Set for another application on Galactic Center imaging data.

⁵ Center for NanoSciences, Ludwig Maximilians University, Munich, Germany

⁶ Rose Fotomasken, Bergisch Gladbach, Germany

easiest one in terms of telescope and instrument design. We have not yet investigated in detail more “exotic” ideas, e.g. illuminating the detectors with a well-defined diffraction pattern or spectrum.

As we see from the discussion in this section, it is crucial to gather the maximum amount of information on the instrumental distortion of MICADO. We therefore conclude that the following steps are necessary:

- Estimating amount and geometrical structure of the distortion theoretically from the optics design.
- Careful mapping of the camera in the lab and at the telescope. Implementation of an internal calibration device, e.g. a calibration mask.
- Testing the system on-telescope for any evolution of the distortion. Evolution parameters can be time (aging effects), telescope orientation (gravitational flexure), etc.
- Additional regular dedicated on-sky calibration observations of sufficient astronomical targets, e.g. star clusters, as secondary tests.

4.3 Telescope Instabilities

Instabilities of the telescope system have the potential to affect astrometric experiments. For the present design of the E-ELT one expects relative intra-night plate scale variations of $\approx 0.1\%$ (Gonzalez (2008)). Across a FOV of $1'$, this corresponds to position variations of order 60 mas. Another effect are adapter-rotator instabilities that can introduce systematic frame-to-frame rotation. Those rotations introduce position misplacements of the same order of magnitude (see, e.g., Trippe et al. (2008) for the case of VLT/NACO).

Fortunately, those effects – shifts, global scalings, rotations – are linear in geometry. Therefore they can be controlled via coordinate transforms of 1st or higher order without additional calibration steps. This statement does not hold however for gravitational flexure effects that introduce a time-variable non-linear instrumental distortion. This phenomenon is covered by our discussion of instrumental distortion in the previous subsection.

4.4 Achromatic Differential Atmospheric Refraction

Any ground-based position measurement is affected by atmospheric refraction. As we discuss relative position measurements of sources located in our FOV, we have to take into account any *relative* or *differential* atmospheric refraction. In this case, we have to discriminate achromatic and chromatic effects.

Achromatic differential refraction is caused by the slight difference in zenith angles ζ of two (or more) sources located within the same FOV. For each source, the atmospheric refraction leads to a deviation between physical and observed zenith angles. For two sources at slightly different zenith angles, those deviations will be different. Therefore the observed distance between the two targets deviates from the physical one and needs to be corrected. The required correction Δx of the distance between two sources 0 and 1 is approximately given by the relation

$$\Delta x = (1 + \tan^2 \zeta_1)(A + 3B \tan^2 \zeta_1)\Delta\zeta. \quad (8)$$

Here ζ_1 is the zenith angle of source 1, $\Delta\zeta$ is the *observed* zenith separation, A and B are constants. Detailed calculations (Gubler & Tytler (1998)) show that the linear terms of this effect are of the order of several milli-arcsec, whereas the quadratic terms are as small as $\approx 1\mu\text{as}$. For example: with $\zeta_1 = 45^\circ$ and $\Delta\zeta = 30''$, the correction amounts to ≈ 15 mas in the first-order terms and $\approx 2\mu\text{as}$ in

second order. Fortunately, these terms can be absorbed by quadratic coordinate transforms (see Sect. 3). From this we conclude that we can neglect achromatic differential atmospheric refraction for the purpose of our analysis.

4.5 Chromatic Differential Atmospheric Refraction

Chromatic differential refraction (CDR) or *atmospheric dispersion* is a more severe problem for accurate astrometry than the achromatic case. As the refractive index n of the atmosphere is a function of wavelength, the observed angular distance between two sources is a function of the (relative) source colours. For a given *true* zenith distance ζ_t , the deviation from the *apparent* zenith distance ζ_a (in radians) follows the approximative relation

$$\zeta_t - \zeta_a \approx R \tan \zeta_t = \left(\frac{n^2 - 1}{2n} \right) \tan \zeta_t \quad (9)$$

Here R is the refraction constant. For standard conditions the refractive index is given by

$$(n - 1) \times 10^6 = 64.328 + \frac{29498.1 \times 10^{-6}}{146 \times 10^{-6} - s^2} + \frac{255.4 \times 10^{-6}}{41 \times 10^{-6} - s^2} \quad (10)$$

with $s = \lambda^{-1}$, λ being the vacuum wavelength in nm (e.g. Cox (2000)). Of course, this has impact on relative astrometry only if the two sources have different colours. For observations of two stars with broadband JHK filters, astrometric errors are of the order of 1 mas (within a wide range, depending on zenith angles, angular distances, and source colours). We therefore need to correct the CDR in order to meet our desired accuracies.

In the following, we investigate the use of an atmospheric dispersion corrector (ADC) placed into the optical path. This ADC is a pair of ZnSe/ZnS biprisms which refracts – for a given zenith angle – the infalling radiation such that the CDR is compensated. In order to use this element for a range of zenith angles, the biprisms can be rotated around the optical axis relative to each other; this controls the strength of refraction. Such a system was built and operated successfully for the SHARP II+ NIR camera system (Eisenhauer (1998)). The required optical components are available in the spatial dimensions needed for MICADO (S. Koebele *priv. comm.*). In order to decide on the design strategy, we explored three options in detail:

- The use of one non-tuneable ADC optimized for simultaneous correction of the full wavelength band range J, H, K_s ($\lambda = 1.1\dots 2.35\mu\text{m}$).
- One ADC covering the entire band range that can be tuned in wavelength to bands J, H, and K_s , respectively.
- Three ADCs, one for each of the three bands J, H, and K_s .

For each option (1, 2, 3), we computed residual dispersions (dispersions left after correction) vs. wavelength for each of the bands J, H, and K_s . All calculations used zenith angles $\zeta = 45^\circ$. Our computations made use of numerical optimization routines implemented into the software package Mathematica⁷. The results for J band are given in Fig. 4. Whereas option 1 results in a very asymmetric curve of correction, options 2 and 3 are very similar in shape and highly symmetric. As we see here, one can reduce the residuals by using an ADC specifically designed for the wavelength band analyzed (option 3) by factors of about ten compared to a tuneable ADC (option 2). However, the impact on astrometry is actually given by the

⁷ Wolfram Research, Inc., Champaign, IL, USA

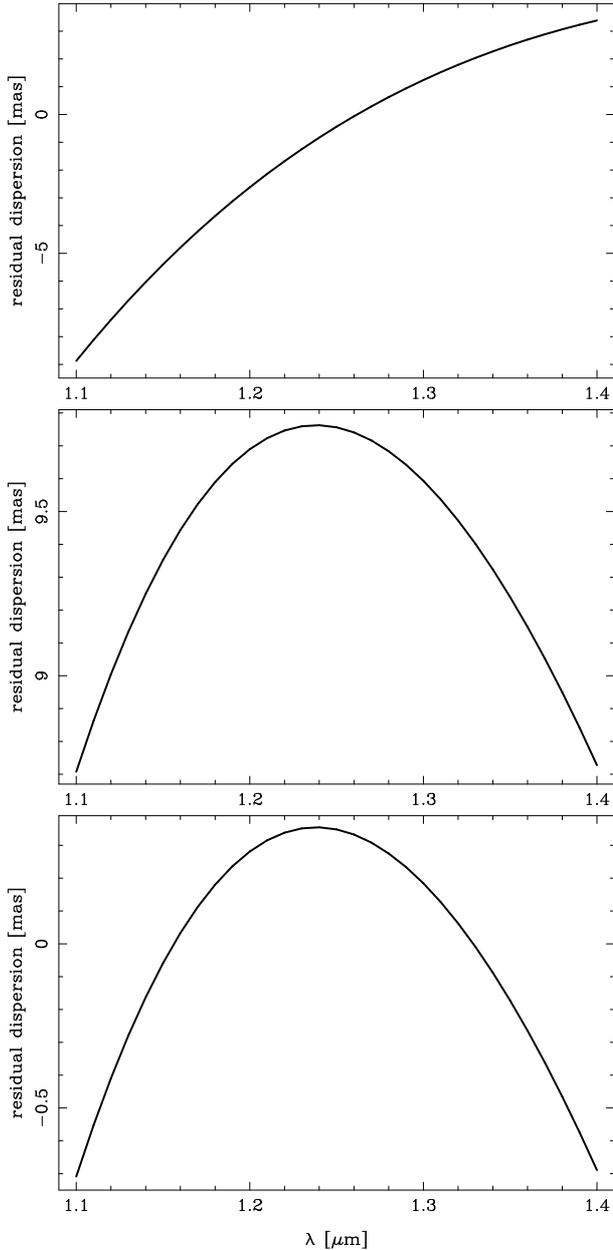


Figure 4. Residual dispersion vs. wavelength in J band after correction with an ADC assuming $\zeta = 45^\circ$. *Top panel:* Using one non-tunable ADC for the entire JHK band range. *Center panel:* Using one tunable ADC for the entire JHK band range tuned to J band. *Bottom panel:* Using an ADC specifically designed for optimum correction in J band. Please note the changes in scales. The non-tunable ADC results in a very asymmetric correction curve. The other two curves are offset by about ten times their amplitudes from each other, but are very symmetric in their shapes.

relative displacement of two sources with different colours. This means that the residual astrometric error is mainly controlled by the symmetry of the curves, not by their amplitudes or absolute levels. We may therefore expect that options 2 and 3 result in very similar astrometric accuracies.

For a quantitative description of the residual astrometric errors we analyzed two hypothetical science cases. We examined

- an observation of two black bodies of very different tempera-

Table 2. Relative zenith angle shifts computed for the options, science cases, and filter bands discussed in the text. All numbers are in μas .

Band	J	H	K _s
Option 1:			
Case A	730	73	196
Case B	642	112	250
Option 2:			
Case A	9	9	6
Case B	35	23	21
Option 3:			
Case A	9	9	2
Case B	35	22	11

tures ($T_1 = 3,000$ K, $T_2 = 30,000$ K), crudely corresponding to M5 and B0 main sequence stars, respectively (“case A”),

- the case of two black bodies at $T = 5,800$ K (e.g. sun-like stars) affected by very different extinctions of $A_V = 25$ and $A_V = 35$, respectively (“case B”). For this case, we used the extinction law by Draine (1989) as taken from Lutz et al. (1996).

For each option, science case, and filter band we computed relative shifts in zenith angle between the sources 1 and 2 via

$$\Delta\zeta_{2-1} = \frac{\int \Delta\zeta(\lambda)S_\lambda d\lambda}{\int S_\lambda d\lambda} \Big|_2 - \frac{\int \Delta\zeta(\lambda)S_\lambda d\lambda}{\int S_\lambda d\lambda} \Big|_1. \quad (11)$$

We present the results of our analysis in Table 2. As we see here, using a non-tunable ADC (option 1) for the entire JHK band range leaves us with errors of roughly $\approx 100\mu\text{as}$ in H band which contains the center of correction. When going to K_s and J bands, the residuals increase to $\approx 200\mu\text{as}$ and $\approx 700\mu\text{as}$, respectively. All numbers exceed any acceptable value by factors of about ten or more. This clearly rules out option 1.

Using a tuneable ADC (option 2) or three ADCs optimized for the three bands (option 3) reduces the residual zenith angle displacements to $\approx 10\mu\text{as}$. This result complies with our requirements and shows that our approach can indeed achieve the necessary accuracies for realistic science cases. As expected from Eqs. 9 and 10, the errors increase with increasing frequency. Therefore one should avoid observations at very short (shorter than J) wavelengths. Additionally, one should consider the use of narrow-band filters in case of (a) observations at short wavelengths or (b) extreme relative source colours.

Another calibration step one should consider is an *a posteriori* correction. As we discuss above, the impact of CDR on astrometric solutions can be quantified analytically if the relative source colours are known. However, this requires careful monitoring of the atmosphere (see, e.g., Helminiak (2009)).

From our analysis of chromatic differential atmospheric refraction, we conclude the following:

- CDR distorts astrometric solutions by up to few mas depending on source colours. We need to implement a dedicated correction.
- Using a tuneable ZnS/ZnSe atmospheric dispersion corrector reduces the astrometric signal caused by CDR to $\approx 10\mu\text{as}$ in JHK bands assuming realistic science cases. We therefore highly recommend to implement such a device into MICADO.
- In case of extreme relative source colours or observations at short (shorter than J-band) wavelengths, one should consider the use of narrowband filters additional to an ADC.

- If the source colours and the atmospheric conditions (temperatures, pressures, humidities) are known with sufficient accuracies, one might additionally apply an analytic *a posteriori* correction of astrometric data.

4.6 Guide Star Measurement Errors

MICADO’s MCAO system is designed to make use of three natural guide stars that are located (at a priori arbitrary positions) in the field of view. Additionally, the use of six laser guide stars is foreseen. The stars are observed simultaneously as references for correcting the wavefront deformation imposed on astronomical signals by the atmosphere. The natural guide stars are used for correcting low-order effects, whereas the laser guide stars correct high-order distortions. The LGS high-order correction does not make use of the guide star positions.

This is different for the NGS low-order correction. This calibration step requires knowledge of the relative guide star positions. The guide star measurements will have finite errors due to atmospheric fluctuations that introduce a “position wander”. Uncertainties in the measured guide star position will introduce time-variable distortion into the AO corrected FOV. For N stars, one may expect distortions up to order $N-1$, i.e. up to 2nd order for the case of three natural guide stars. These distortions will differ from image to image. Therefore it is necessary to combine images with full coordinate transform of minimum order two. If this is done, the effect will be compensated completely.

4.7 Differential Tilt Jitter

The light from a science target and the light from an adaptive optics system reference source travel through different columns of atmospheric turbulence. An AO system applies a tip-tilt correction to the signal received from the AO reference source. This correction is slightly different for other positions in the field of view. Therefore, any two objects in the observed field suffer from *differential tilt jitter*: a random, achromatic, anisotropic fluctuation of the observed angular distance of the two sources (e.g., Britton (2006); Cameron et al. (2009)). In first order, the rms of this fluctuation follows the relation

$$\sigma_{TJ} \propto \theta \times D^{-7/6} \times \left(\frac{\tau}{t}\right)^{1/2}. \quad (12)$$

Here θ is the angular distance between the sources, D is the telescope aperture, τ is the aperture wind crossing time (approximately: D divided by wind speed), and t is the integration time. Tilt jitter is an anisotropic effect with

$$\sigma_{\parallel} = \sqrt{3}\sigma_{\perp} \approx 1.732\sigma_{\perp} \quad (13)$$

where σ_{\parallel} , σ_{\perp} denote the tilt jitter rms parallel and perpendicular to the line connecting the two sources, respectively.

In Fig. 5 we give an example for a tilt jitter signal observed in images taken with an 8-m-class telescope. This result is from the work by Fritz (2009) who analyzed diffraction-limited VLT images of the nuclear cluster of the Milky Way. For pairs of stars, he computed the uncertainties of the measured distances parallel and perpendicular to the lines connecting the two stars. Fig. 5 shows the uncertainty as function of star angular distance. By means of linear fits one finds a relation $\sigma_{\parallel}/\sigma_{\perp} = 1.91 \pm 0.22$, i.e. $\sqrt{3}$ within errors – as expected from Eq. 13. From the line slopes and the known integration times one can estimate an aperture wind crossing time of $\tau \approx 0.6$ s. For details refer to Fritz (2009) and Fritz et al. (2009).

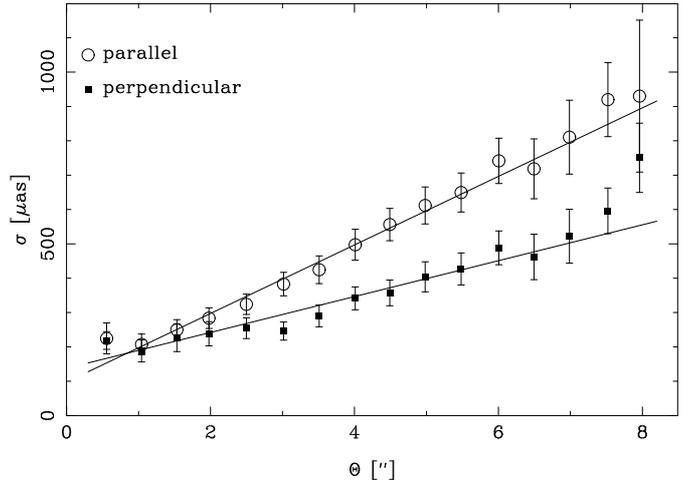


Figure 5. A differential tilt jitter signal as observed in VLT images. This diagram shows the error on the measured distance between two stars σ as a function of star angular distance θ . Errors are calculated parallel (\parallel) and perpendicular (\perp) to the line connecting the two stars on sky. Points with errorbars are data, continuous lines indicate the best fitting linear models. From the line slopes one finds $\sigma_{\parallel}/\sigma_{\perp} = 1.91 \pm 0.22$, i.e. $\sqrt{3}$ within errors as expected from Eq. 13. This result is taken from Fritz (2009).

The impact of this effect on astrometry can be substantial. To give a reference: based on observations with the Hale 200 inch telescope, Cameron et al. (2009) find astrometric uncertainties of ≈ 75 mas for $D = 5$ m, $\theta = 60''$, and $t = \tau = 0.2$ s (i.e. $\tau/t = 1$). For a realistic intra-epoch E-ELT observation with $D = 42$ m and $t = 100\tau \approx 100$ s (i.e. $\tau/t = 0.01$), this scales to $\sigma_{TJ} \approx 600\mu\text{as}$. Averaging out this error to reasonable scales (less than $50\mu\text{as}$) would take about four hours. This imposes severe constraints on any astrometric observation.

The calculation given above is strictly valid only for single-conjugate AO (SCAO) systems, i.e. AO systems using one reference source. However, MICADO is designed to use an MCAO system with three natural guide stars initially. The use of multiple guide star should reduce the tilt jitter error substantially. Ellerbroek (2007) finds that for a 30m telescope and the case of three guide stars arranged in an equilateral triangle the error is reduced by a factor ≈ 6 compared to the SCAO case. Scaling the results by Ellerbroek (2007) to the case of the E-ELT, one finds

$$\sigma_{TJ} \approx 430 \times t^{-1/2} \mu\text{as} \quad (14)$$

with t being the integration time in seconds and $\theta = 60''$. This means that accuracies of $\approx 10\mu\text{as}$ can be achieved within integration times of about 30 minutes.

A helpful property of tilt jitter is the fact that it is random in time, but correlated in space. This is also reported by Fritz (2009) who finds that already using linear coordinate transforms between images reduces the error by a factor ≈ 2 . According to Cameron et al. (2009), the use of coordinate reference frames based on weighted pairwise distances between a target source and several reference sources is able to catch the tilt jitter error. This approach is similar to the concept of kriging (e.g. Clark & Harper (2000)). It requires some tens of astrometric reference points in the FOV in order to obtain a sufficient number of pairwise baselines. Additionally, it requires any intra-epoch dataset to consist of some tens of individual exposures instead of few long-term integrations. This is necessary in order to estimate the uncertainty (and thus the weight) of each pairwise source distance from its histogram. If this scheme

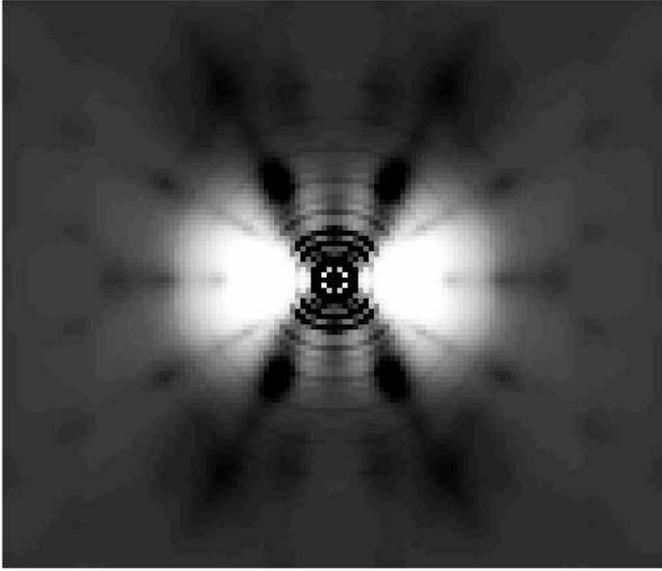


Figure 6. Difference image of two simulated MAORY K-band PSFs located $70''$ away from each other at opposite sides of the center of AO correction. This image covers a field of $\approx 7.5'' \times 6''$. The difference map shows a complicated, highly symmetric pattern.

is applied, the error can be reduced to $\approx 10\mu\text{as}$ within few minutes of integration.

From our discussion on differential tilt jitter we conclude for MICADO:

- Differential tilt jitter can introduce errors of order $\approx 100\mu\text{as}$ into typical E-ELT/MICADO observations. This effect is not a challenge for the instrument design but for the planning of observations and data calibration.
- The tilt jitter error integrates out with time as $\propto t^{-1/2}$. For the E-ELT, reducing the error to order of $\approx 10\mu\text{as}$, integration times of at least ≈ 30 min per intra-epoch dataset will be necessary.
- The tilt jitter error can be calibrated out (down to $\approx 10\mu\text{as}$) by using dedicated astrometric reference frames and transforms. This requires (a) some tens of reference sources in the FOV and (b) some tens of individual exposures per intra-epoch dataset.

4.8 Anisoplanatism

All existing AO systems suffer from *anisoplanatism*, meaning that the shape of a PSF is a function of its position in the field of view. Usually, this spatial variability is described by the relation

$$P(\theta) = P(0) * K(\theta) \quad (15)$$

(Fusco et al. (2000); Steinbring et al. (2002; 2005); Cresci et al. (2005)). Here θ is the angular distance from the center of AO correction, $P(0)$ [$P(\theta)$] is the PSF at the center of correction [at distance θ], and $K(\theta)$ is a kernel describing the PSF variation; * denotes convolution.

If $K(\theta)$ were asymmetric, then the center of light of the PSF – and thus the detector position – would be a function of θ . This would introduce a systematic distortion of astrometric solutions that has to be calibrated out.

For existing SCAO systems, $K(\theta)$ is known to be symmetric. It can be approximated analytically as an elliptical Gaussian profile (e.g. Steinbring et al. (2005)). In this case, the most important

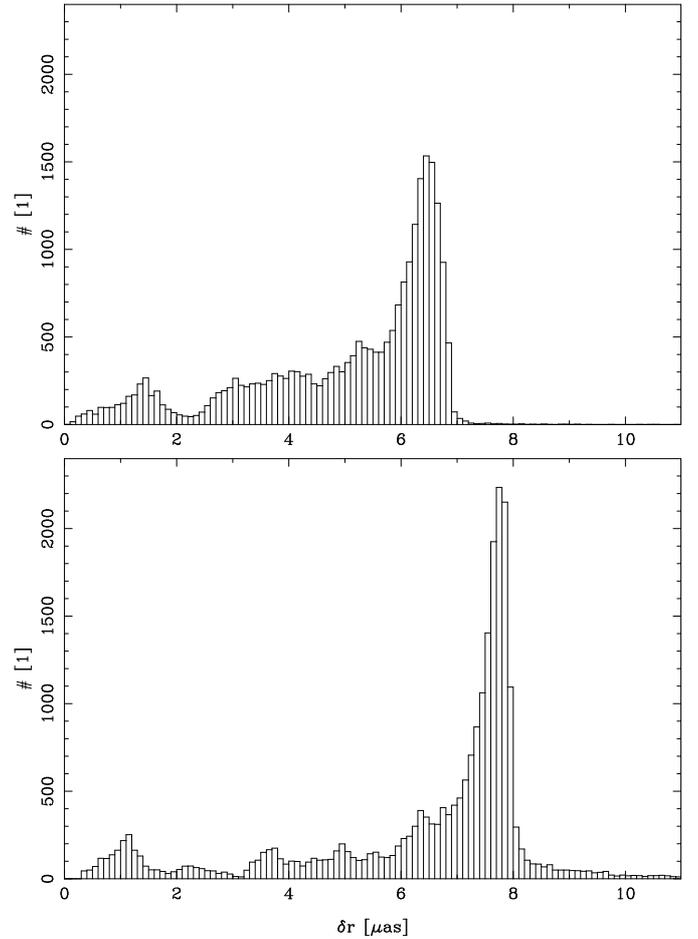


Figure 7. Astrometric errors introduced by MCAO anisoplanatism in H (*top panel*) and K (*bottom panel*) bands. Errors are centroid shifts (with respect to the case of perfectly symmetric light distributions) of the difference images between pairs of simulated PSFs located at different locations in the MAORY FOV. The histograms have cutoffs at $\approx 7\mu\text{as}$ in H band and at $\approx 8\mu\text{as}$ in K band.

impact of anisoplanatism is a systematic degradation of the PSF's Strehl ratio S like

$$S(\theta) = S_0 \exp[-(\theta/\theta_N)^{5/3}] \quad (16)$$

assuming a perfect AO correction. Here S_0 is the Strehl ratio at the center of AO correction, the parameter θ_N is the anisoplanatic angle. This effect reduces the SNR of a source and thus the statistical astrometric accuracy, but a priori does not introduce a systematic error.

For the case of MCAO systems with multiple centers of correction the problem is less well understood. In order to estimate the astrometric error introduced by anisoplanatism, we made use of preliminary simulations of multi-guide-star MAORY PSFs in H and K bands⁸. The simulation data provide a grid of 217 PSFs located at distances between $0''$ and $80''$ from the center of correction. The simulations assume a seeing of $0.6''$. PSFs are sampled with pixel scales equal to $\lambda/2D$, corresponding to 4.05 mas/pix and 5.30 mas/pix for H and K bands, respectively.

⁸ Made available by the MAORY consortium at <http://www.bo.astro.it/~maory/Maory/>

By construction, each PSF is centered accurately at the central pixel of a 512×512 pixel grid. As discussed above, we need to estimate the relative position error introduced by anisoplanatism. For this we computed for each pair of PSFs a difference map by subtracting one PSF image from the other. In Fig. 6 we show the difference image for the case of two PSFs located $70''$ away from each other at positions $(-60'', 0'')$ and $(+10'', 0'')$ with the center of AO correction located at $(0, 0)$. Clearly, the map shows a complicated, highly symmetric light distribution. From the high degree of symmetry one can already suspect qualitatively that the impact of MCAO anisoplanatism on the astrometry is small.

In order to quantify this statement, we calculated for each difference map the centroid

$$x_{\text{centroid}} = \frac{\sum_i x_i \cdot I_i}{\sum_i I_i} \quad (17)$$

of the light distribution. Here x is an arbitrary coordinate, x_i and I_i are position and flux assigned to the i -th pixel. The summation is performed over all map pixels (e.g. Berry & Burnell (2000)). As we had 217 PSF images at hand, this procedure resulted in 23436 measurements. Centroid positions different from the center of the pixel grid correspond to position shifts of one PSF with respect to the other caused by asymmetries in the anisoplanatism kernel.

We summarize our results in Fig. 7. The histograms of the position shifts show quite sharp cutoffs at $\approx 7\mu\text{as}$ in H band and at $\approx 8\mu\text{as}$ in K band. We do not find a significant correlation of position errors with distance from the center of correction or any other systematic relation.

We conclude that MCAO anisoplanatism probably introduces very small but noticeable uncertainties into astrometry. Depending on wavelength band we find errors (upper ends of histograms) up to $\approx 8\mu\text{as}$. As the anisoplanatism effect is not (or very weakly) correlated with PSF positions in the FOV, it can probably not be caught by coordinate transforms or any other systematic correction but has to be included into the error budget.

4.9 High- z Galaxies As Reference Sources

Throughout this article we discuss *relative* astrometry, meaning measuring relative source positions. So far we implicitly assumed the reference sources to be point sources, especially stars. However, in some cases the use of reference points other than stars will be necessary:

- The number of stars located in the FOV can be too small. As discussed in Sect. 3, the number of reference sources is a function of the order of the coordinate transform used. For example, in case of a 2nd-order transform, the minimum number is six. Using a larger number than the theoretical minimum is useful to average out statistical position errors that could propagate into the transformation. Additionally, systematic effects like instrumental distortion (Sect. 4.2) and differential tilt jitter (Sect. 4.7) require large numbers of reference points for calibration purposes. Deep fields at moderate galactic latitudes ($|b| \approx 50^\circ$) include only a few to a few tens $K_{\text{AB}} < 23$ stars/arcmin (see e.g. Förster Schreiber et al. (2006); Wuyts et al. (2008)). This number is probably too small for astrometric MICADO observations.

- The science case might require an extragalactic reference frame, thus excluding Galactic stars. For example, several groups (e.g. Bedin et al. (2003; 2006); Kalirai et al. (2004)) use samples of background galaxies as reference in order to measure the absolute

motion of globular clusters. Other science cases may need similar approaches (see Sect. 2).

In order to investigate the use of galaxies as references, we analyzed two simulated MICADO deep field K-band images astrometrically. We simulated galaxies following realistic distributions of K-band magnitude as function of redshift z taken from the Chandra Deep Field South FIREWORKS (“field A”; Wuyts et al. (2008)) and FIRES Hubble Deep Field South (“field B”; Labbé et al. (2003)) catalogs. All objects in the fields are built using four main ingredients:

- Smooth elliptical and disk galaxy light distributions with realistic surface number densities, sizes, and K- z distributions. The K- z distributions were obtained by drawing at random galaxies from the two source catalogues above, scaling the numbers by the ratio of the MICADO FOV and the respective survey areas. A fixed mix of ellipticals and disks was adopted (40%/60%), and effective radii were assigned at random so as to match approximately the size distributions of Franx et al. (2008). Uniform distributions are assumed for the axis ratios (with minimum of 0.3 for ellipticals and 0.1 for disks) and for the position angles.

- Bulges (for disk galaxies) with de Vaucouleurs luminosity profiles adopted for simplicity. Bulge properties are poorly constrained at high redshift; our assumptions were guided by results from Elmegreen et al. ((2009), and references therein; see also Genzel et al. (2008)). We assumed uniform distribution of axis ratios between 0.7 and 1, position angles fixed at those of the host disks, effective radii inversely proportional to $1+z$ and such that $r_e = 2\text{kpc}$ at $z = 0$, and a random distribution of bulge-to-total light ratios between 0 and 0.5.

- Clumps (for disk galaxies) with realistic numbers per disk, typical light fractions, characteristic sizes and dependence on redshift, and radial distribution across the disks so as to roughly match observed properties at $z \approx 1-2.5$ (e.g., Genzel et al. (2008); Elmegreen et al. (2009) and references therein) and their lack at $z \approx 0$. Of order 1–10 clumps per disk were simulated, with Gaussian light profiles (again for simplicity), a narrow distribution of FWHMs varying as $(1+z)$ such that $\text{FWHM} = 1\text{ kpc}$ at $z = 2.5$, light fractions of a few percent typically, an exponential distribution of radial positions with scale-length five times that of the host disk, random position angles and axis ratios between 0.7 and 1.

- Unresolved star clusters (for all galaxies). This is the ingredient which is least constrained at high redshift, as current instruments lack the resolution and point-source sensitivity to detect individual clusters if such exist at high z . To define their properties, we assumed superpositions of five different populations of point-like sources with a range of (Gaussian) magnitude and radial distributions. The brightest population contains the fewest clusters, with the most centrally concentrated distributions. The brightest cluster in each galaxy contributes 0.1% of the total light. These assumptions roughly reproduce those of super star clusters and globular clusters in local galaxies, and unresolved sources in local analogs of $z \gtrsim 3$ Lyman-break galaxies (Overzier et al. (2008)).

For each field image we created three realizations corresponding to E-ELT/MICADO integration times of 1, 4, and 10 hours, respectively, by adding Gaussian noise to the original map. As given by the input catalogs, field A has a point source brightness limit of $K_{\text{AB}} = 24.3$, field B of $K_{\text{AB}} = 25.6$. These limits are 5σ depths for point sources in circular apertures of $2''$ diameter. In Fig. 8 we show four simulated galaxies from field B as examples.

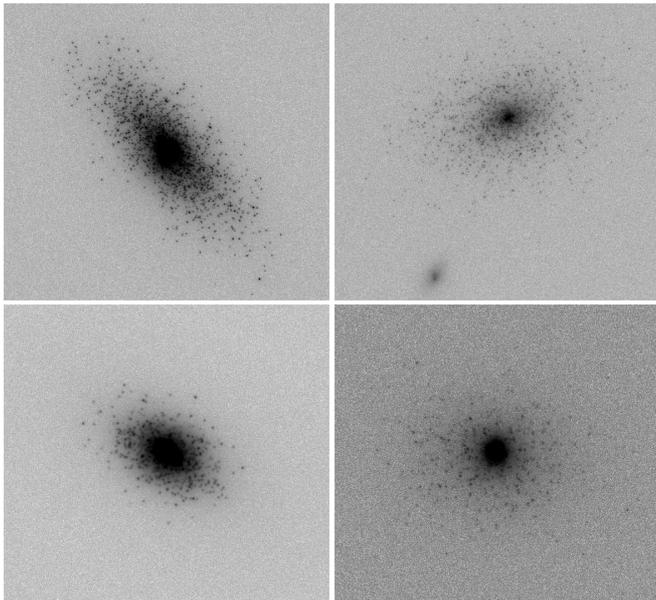


Figure 8. A few simulated galaxies from our artificial deep field B. Images in the top line cover $\approx 3.5'' \times 3.5''$, images in the bottom line cover $\approx 1.8'' \times 1.8''$.

The star clusters are especially interesting for astrometry. They provide an ensemble of several thousands of bright, point-like sources that can be analyzed with standard tools developed for point source astrometry. In each field we selected all star clusters that were sufficiently isolated, meaning a distance of at least 30 mas (about three resolution elements) to the next source. In total we used 1600 sources in field A and 2174 sources in field B. In each image we fit the clusters with 2D Gaussian brightness distributions in order to accurately (meaning few milli-pixels in the best cases) determine their positions. We then compared the results with the true source positions on the original noise-free maps. The distributions of the deviations between true and measured positions provide measures of astrometric accuracies.

We show the results for 10h integration times in Fig. 9. For both fields we find distributions with 1σ -widths of $\approx 920\mu\text{as}$ (field A) and $\approx 650\mu\text{as}$ (field B) per coordinate. These numbers correspond to the typical measurement accuracies for individual star clusters. The global astrometric accuracy is given by the standard error of the mean of the distribution, i.e. σ/\sqrt{N} with N being the number of sources.

We summarize the astrometric accuracies found for all fields and integration times in Table 3. Not surprising, we find the highest accuracies of $23\mu\text{as}$ and $14\mu\text{as}$ (per coordinate) for fields A and B, respectively, for the largest integration time of 10h. The errors increase to about $65\mu\text{as}$ and $29\mu\text{as}$ at integration times of 1h. The systematic difference between fields A and B originates from their different depths: field B is based on a catalog that has a ≈ 1.3 magnitudes deeper brightness limit; it is therefore “richer” of sufficiently bright targets. As MICADO will perform astrometric observations of sources $K_{AB} < 26$ (see also Sect. 1), the results found from field B are actually more realistic for describing the performance of MICADO. We can therefore conclude that total (i.e. the quadratic sum of the values for both coordinates) astrometric accuracies of $\approx 20\mu\text{as}$ are realistic for integration times of order 10h when using galactic star clusters as references.

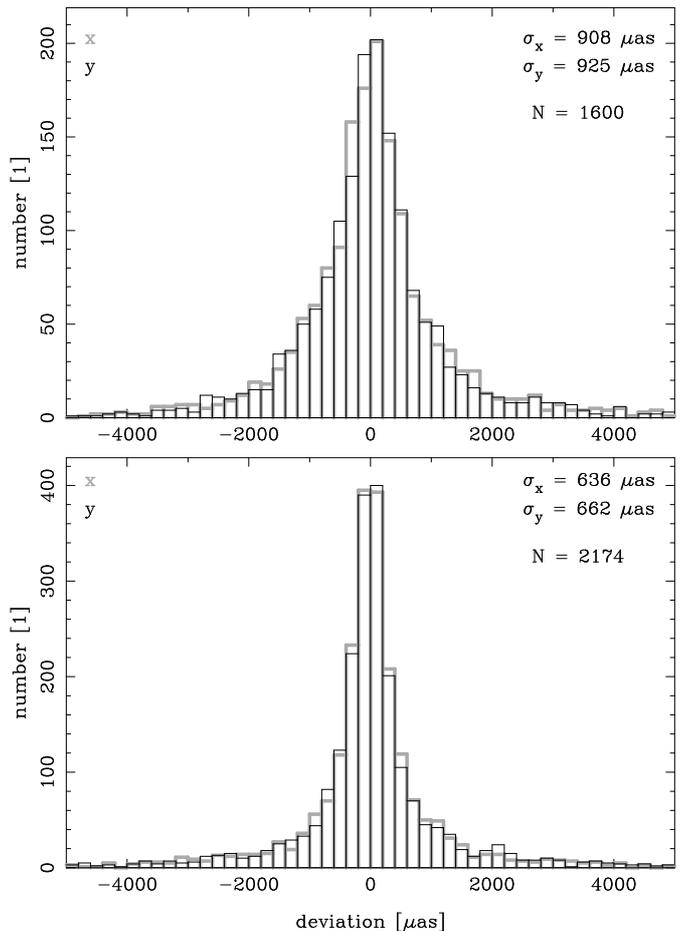


Figure 9. Histograms of position errors found from using several thousand point-like star clusters as references. These results are for fields A (top panel) and B (bottom panel) at 10h integration time. Please note the different number axes scales.

So far, we have taken into account non-resolved (point-like) sources as references only. However, the global extended light distribution contains additional information. We therefore computed 2D cross-correlations between different realizations of the same “observations”. As reference we used one 7000×7000 pixels ($21'' \times 21''$) sub-field of field A containing seven galaxies. For each integration time (1h, 4h, 10h) we computed three realizations of the random noise map added to the original light distribution. For all pairwise combinations we computed the 2D cross-correlations. We calculated their centers by means of 2D Gaussian fits to the central parts of the cross-correlation maps. The uncertainty of the map centers provides a measure of astrometric accuracy. For all integration times we found very similar results: $4.5\mu\text{as}$ at 1h, $4.1\mu\text{as}$ at 4h, and $3.9\mu\text{as}$ at 10h. This indicates that at these level the accuracy is only weakly correlated with the SNR but dominated by systematic effects. In any case the impressive accuracies of $\approx 4\mu\text{as}$ suggest that using 2D cross-correlations of images can provide highly accurate astrometry.

However, 2D cross-correlations cannot be used for relative astrometry in a straight forward manner. By construction, cross-correlations are only sensitive to shifts between images. They cannot describe transformations like scalings, rotations, or higher order transforms as discussed in Sect. 3. Using them for astrometry therefore requires a more sophisticated approach than computing the

Table 3. Astrometric accuracies found from using several thousand point-like star clusters as references. We give errors for the coordinates x and y as function of field and integration time.

	field A		field B	
t_{int} [h]	δx [μas]	δy [μas]	δx [μas]	δy [μas]
10	23	23	14	14
4	30	36	17	17
1	58	65	29	29

cross-correlation between two (or more) images. One ansatz might be to calculate cross-correlations for a sufficient number (about ten or more) of sub-fields of the target area. This would provide a set of relative position (changes). Feeding this dataset into a proper model might result in a highly accurate coordinate transform.

From our analysis we conclude that the use of high- z galaxies as astrometric calibrators is feasible at the level of $\approx 20\mu\text{as}$ at integration times of about 10h. This number assumes the use of several thousand point-like star clusters as reference sources (also assuming that such exist at intermediate to high redshift). Cross-correlation of images might provide even better accuracies, but would require a sophisticated computational scheme that can actually provide full coordinate transforms.

In some cases, one might consider separating the intra-epoch and the inter-epoch calibration steps. Firstly, one can use a small number of stars (as discussed earlier, six may be sufficient) for *intra*-epoch (frame-to-frame) calibration. Proper adding of images allows building up SNR sufficient for detection of and position measurements on faint galaxies. Secondly, one can use these galaxies for *inter*-epoch calibration.

4.10 Calibration of the Projected Pixel Scale

In Section 3 we have outlined the concepts of relative astrometry and coordinate transforms. These schemes make use of sets of reference positions $\{\mathbf{x}_{\text{ref}}^n\}$ in order to compute transformation matrices for the n -th dataset. However, all transformation and calibration steps we have discussed up to now are executed in image space, i.e. in units of pixels. After removing non-linear distortions from the data, one needs to calibrate the pixel scale *as projected on sky* in order to accurately convert measured positions and motions into angular units. We note that the projected pixel scales for the two coordinates x, y may be different if the detector plane is tilted with respect to the focal plane.

Calculating the scaling factors requires astrometric reference points with known positions located in the target FOV. The number of reference sources should be at least three in order to allow for a full linear transformation. Fortunately, this calibration step needs to be executed only once for a selected “master” (or “zero”) image. All other images can be connected to the master image reference frame via coordinate transforms, including the proper scaling (e.g. Trippie et al. (2008)). Errors on the reference positions propagate into the positions and motions calculated from the data.

There are several possibilities for obtaining very accurate reference source positions. One of them is the use of sources located in the MICADO FOV and visible in both NIR and radio, e.g. QSOs or maser stars. If radio-interferometric positions (e.g. VLBI) – which are accurate on the sub-mas level – are at hand, the uncertainty on the pixel scale can be very low (see, e.g., Reid et al. (2007) for the

case of the Galactic center). Another option is the use of stars from the HIPPARCOS catalogue which provides a median position accuracy of $\delta x \approx 1 \text{ mas}$ (Perryman et al. (1997)). One should also consider data from the future GAIA astrometry space mission for calibration as soon as they become available. For the FOV of MICADO, reference position accuracies of order 1 mas translate into a *relative* scaling accuracy of

$$\delta x/x \approx 1\text{mas}/53'' \approx 2 \times 10^{-5}$$

As discussed in Section 2, the science cases for E-ELT/MICADO demand accurate measurements of positions and proper motions over spatial scales from few ten μas (e.g. parallaxes of globular clusters; see Sect. 2.3) to few hundred mas (e.g. stellar orbits around Sgr A*; see Sect. 2.1). A relative pixel scale accuracy of 2×10^{-5} corresponds to an error of 10 μas over an angular distance of 500 mas. We therefore conclude that the calibration of the projected pixel scale introduces errors of $\approx 10\mu\text{as}$ at most.

5 RESULTS AND ERROR BUDGET

We have identified and discussed ten effects that might limit the expected astrometric accuracy of E-ELT/MICADO observations systematically. We have been able to quantify each of these sources of error. From this, we can calculate a prediction for the error budget of MICADO.

(i) For isolated point sources, *detector sampling / binning* does not introduce noticeable ($\approx 1\mu\text{as}$) errors as long as the pixel scale does not exceed 3 mas/pix. For sources affected by crowding, using a smaller scale of 1.5 mas/pix can improve the accuracy by factors of about two compared to the 3 mas/pix case. Therefore MICADO will use a pixel scale of 3 mas/pix as standard and a reduced scale of 1.5 mas/pix for mapping crowded fields. For the error budget, we can thus assume a sampling error

$$\sigma_{\text{samp}} = 1 \mu\text{as} .$$

(ii) Instrumental *geometric distortion* needs to be taken into account by dedicated calibration procedures. We propose to implement a calibration mask into MICADO that illuminates the detectors with a well-defined image. Such a mask would have to be mapped with accuracies of $\approx 40\text{nm}$. Based on our results in combination with published works using Hubble Space Telescope data, we estimate that distortion can be corrected down to levels of ≈ 10 – $30\mu\text{as}$. For the error budget, we therefore use

$$\sigma_{\text{dist}} = 30 \mu\text{as} .$$

(iii) *Telescope instabilities*, notably plate scale instabilities and instrumental rotations, are linear effects that can be absorbed by coordinate transforms. Therefore they do not contribute to the error budget.

(iv) Atmospheric *achromatic differential refraction* is important (order 10 mas) only in linear terms which can be absorbed by coordinate transforms. Higher-order contributions are of order $1\mu\text{as}$, meaning for the error budget

$$\sigma_{\text{ADR}} = 1 \mu\text{as} .$$

(v) *Chromatic differential refraction* introduces position errors of the order ≈ 1 mas in NIR observations depending on relative source colours. A tuneable ZnS/ZnSe atmospheric dispersion corrector can reduce this effect to $\approx 10\text{--}20\mu\text{as}$ for “typical” science cases. Extreme relative source colours might require the additional use of narrowband filters and/or analytic a posteriori correction schemes. For the error budget, we thus set

$$\sigma_{\text{CDR}} = 20 \mu\text{as} .$$

(vi) *AO guide star measurement errors* for N natural guide stars can introduce distortions up to order $N-1$ into images, meaning 2nd order distortions for the three NGS of MICADO. This effect can be absorbed by coordinate transforms (of order $N-1$). It therefore does not contribute to the error budget.

(vii) *Atmospheric differential tilt jitter* can introduce errors of $\approx 100\mu\text{as}$ into diffraction-limited E-ELT observations. It integrates out with $t^{-1/2}$. For MICADO which uses an MCAO system, the tilt jitter error can be integrated down to $\approx 10\mu\text{as}$ within about 30 minutes of observation. Using dedicated coordinate transform allows reaching this accuracy in shorter times. For the error budget, we thus set

$$\sigma_{\text{TJ}} = 10 \mu\text{as} .$$

(viii) *The anisoplanatism* of the MAORY AO system introduces uncertainties of up to $\approx 8\mu\text{as}$. There appears to be no correlation of this error with the position of a PSF in the FOV; it therefore can not be calibrated out in a straight forward manner. Therefore we add it to the error budget:

$$\sigma_{\text{aniso}} = 8 \mu\text{as} .$$

(ix) Depending on target field and science case, the use of *galaxies as astrometric calibrators* may be necessary. From a simulated MICADO deep field we find that we can use several thousand non-resolved galactic star clusters as point-like reference sources. However, good accuracies of $\approx 20\mu\text{as}$ require long intra-epoch integration times of about 10 hours; we consider this to be a somewhat large but realistic timescale. We therefore use for the error budget

$$\sigma_{\text{galaxies}} = 20 \mu\text{as} .$$

(x) The accuracy of the *sky-projected pixel scale* is limited by the accuracy of astrometric positions of reference sources in the FOV. Given the typical accuracies of present-day catalogues which are of order 1 mas, we add to the error budget

$$\sigma_{\text{scale}} = 10 \mu\text{as} .$$

From the individual uncertainties listed above we can calculate a total intrinsic astrometric accuracy for MICADO as

$$\sigma_{\text{sys}} = \sqrt{\sum_i \sigma_i^2} = 44\mu\text{as} . \quad (18)$$

This number provides a systematic limit for astrometric accuracies to be expected from MICADO data. Of course, this result corresponds to a somewhat arbitrary “typical” case. As many parameters like integration times, source colours, numbers and types of reference sources, etc. can vary over wide ranges, the actual σ_{sys} for a specific observation can be quite different – in both directions –

from the one we quote here. Nevertheless, we conclude that we are able to quantify the mean systematic astrometric accuracy achievable with MICADO which is $\approx 40\mu\text{as}$.

When discussing the accuracy of the measurement for a given target, one of course needs to add the statistical measurement error σ_{L} (Eq. 5) which scales with the SNR. For $\text{SNR}=100$, $\sigma_{\text{L}} = 34\mu\text{as}$, and thus the combined error is $\sqrt{\sigma_{\text{sys}}^2 + \sigma_{\text{L}}^2} = 56\mu\text{as}$. SNRs different from 100 modify this number accordingly.

6 CONCLUSIONS

In this article we have studied the capabilities expected for the NIR imager MICADO for the future 42-m European Extremely Large Telescope with respect to accurate astrometry. A variety of science cases requires long-term astrometric accuracies of $\approx 50\mu\text{as}$. We discuss and quantify ten effects that potentially limit the astrometric accuracy of MICADO. We conclude that the systematic accuracy limit for astrometric observations with MICADO is $\sigma_{\text{sys}} \approx 40\mu\text{as}$. We find that astrometry at this accuracy level with MICADO requires the fulfillment of several conditions:

- All images, regardless of their distance in time, need to be combined via full coordinate transforms of second order or higher.
- MICADO needs to be equipped with an astrometric calibration mask for monitoring the instrumental distortion. The pixel scale of the camera should not exceed the 3 mas/pix used in the current design.
- Astrometric observations require decent integration times of at least 30 minutes per epoch. This is unavoidable in order to average out atmospheric tilt jitter. When using high- z galaxies as astrometric reference points, integration times up to about 10 hours can be necessary.

It is noteworthy that the effects discussed in this article already affect observations collected with present 8m-class telescopes. In his exhaustive analysis of NIR images obtained with VLT/NACO, Fritz (2009) has been able to detect signatures of chromatic differential refraction and differential tilt jitter in his astrometric dataset. He concludes that taking into account these effects can improve the accuracies down to few hundred μas . This agrees with the findings of Lazorenko (2006) and Lazorenko et al. (2007) who analyze seeing-limited optical R-band ($\lambda_{\text{center}} = 655\text{nm}$) images taken with VLT/FORS1+2. They conclude that they are able to achieve astrometric precisions (but not accuracies) of $\approx 100\mu\text{as}$ by using a special scheme for scheduling observations and dedicated coordinate transforms (although they neglect instrumental geometric distortion).

The analysis we provide here is set up for the specific case of E-ELT/MICADO, but parts of our results are valid in general. This study should thus contain valuable information for other future 30–40m telescopes. As some of the effects we discuss are actually observed in present day 8m-class telescope data, our analysis might also be helpful for the calibration of data already taken. We therefore expect that our work is of interest well beyond the E-ELT community.

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